Volatility Modeling and Asset Pricing: Extension of
GARCH Model with Macro Economic Variables,
Value-at-Risk and Semi-Variance for KSE

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Abstract
The purpose of this study is to identify the behavior of returns and volatility with the
attributes of non-linearities and asymmetric patterns in the returns series of KSE and
modeling of volatility for asset pricing with macroeconomic, value at risk and semi-
variance in GARCH specification. Daily data is used for stock returns for the period of
Jan 2000 to Dec 2015 and the data for macroeconomic variables is taken for the period
Jan 2000 to Dec 2015 on monthly basis. GARCH and GARCH-in-mean model is used
for modeling the volatility in this study. GARCH-in-mean model is extended with
macroeconomic variables in mean as well as with variance equation and findings indicate
that the change in interest rate has significant negative effect on returns and increase in
change in interest rate will decrease stock returns. The changes in oil prices have also
significant positive impact on KSE returns. The results indicate that interest rate is
significantly positively related to volatility. However oil prices change has also negative
significant impact on volatility. It is concluded that macroeconomic variables are
significant parameters for explaining the stock returns as well as volatility. Further
GARCH (1, 1) Model is extended with Value at Risk in mean and variance equation. It is
concluded that VaR is significantly negatively related to the returns of KSE market in
GARCH specifications. Moreover GARCH (1,1) Model is extended with the Semi-
variance for KSE. It is concluded that semi-variance is significant and indicates that
downside risk has negative impact. In last study provides evidence that volatility
influences returns in a non-linear fashion. This study provides an insight about the
behavior of risk and return in emerging market of Pakistan which is prime area of interest
for investors.

Keywords: GARCH-in-Mean, macroeconomic variables, conditional volatility, value-
at-risk, semi-variance, asymmetric patterns.
1. Introduction
Volatility and asset pricing in financial markets is the most important area in the field of financial economics in the present era. Volatility or variance is generally considered as risk. The risk that lies in returns is measured through the corresponding variance, and hence investors should hold an efficient portfolio by means of mean-variance with the maximum possible return along with given level of variance. The behavior of returns and volatility in emerging markets is always a matter of interest and Pakistan is no exception. Non-linearities and asymmetric pattern in the returns and volatility in emerging markets are unique attributes of these markets. Emerging markets have higher volatility and produce higher returns and macroeconomic variables and other risks factors may play a dynamic role in such economies for the movement of returns and volatility. So asset pricing in the presence of such behavior is still an unaddressed issue in the Pakistan economy. Therefore this study is an effort to probe into the matter for the induction of conditional volatility and non-linearities perspective in an asset pricing model.

Investors who are risk averse and therefore they require an additional premium as compensation in order to hold a risky asset. It is a positive function of the risk to attain higher premiums because the higher the risk then higher the premium should be. On the other hand if the risk is detained by the volatility parameter or by the conditional variance, then the conditional variance may be the conditional mean function as well.

Efficient Market Hypothesis Fama (1970) put the fair game model and this theory assists to identify that volatility follows the hypothesis of random walk or not in an auto regressive process and either prediction of volatility is possible or not. Whereas volatility theories include Leptokurtic, Volatility clusters or volatility pooling and leverage effect behavior in financial time series. In short Brooks (2008) identified that a very few number of non-linear models are useful for modeling the financial data. The ARCH model of Engle (1982) and the GARCH model of Bollerslev (1986) have got lots of support, and are used in the development of these models for the conditional variance. As these models got popularity and established a popular tool for modeling and forecasting the financial time series.

The most famous non-linear financial models are ARCH or GARCH models used for modeling and forecasting volatility, hence switching models, which permit the behavior of a financial time series to back up various processes at different points in time. A non-linear model should be applied where financial theory proposes that the relationship between variables should be such which requires a non-linear model, here the most important query is that which tools are available to identify non-linear behavior in financial time series. There are number of tests for non-linear patterns in time series that are available to the researcher. While studying asymmetric patterns in mean and variance support is required to a distribution that can handle these irregularities and to determine asymmetric models for mean and variance. These theories motivates for this study on the grounds that such element have not yet been explored with extended parameters in the markets to be studied. Asset Pricing theories evolved from Capital asset pricing model (CAPM), originally introduced by Sharpe (1964) and Lintner (1965) based upon the mechanics of mean, variance optimization in Markowitz (1952). Thereafter Chen Roll Ross(1986) introduced macroeconomic based risk factor model. Fama and French (1993) introduced SMB and HML in extension to CAPM in a particular microeconomic based risk factor model. Carhart (1997) extended the Fama French three
factor model by including a fourth common risk factor of momentum factor and estimated it by taking the average return to a set of stocks with the best performance over the prior year minus the average return to stock with the worst returns. Volatility theories and asset pricing theories can be extended in a new modeling approach of conditional volatility and asset pricing. GARCH family models can be extended for risk return relationship along with macroeconomic models to have a superior look.

The objective of this study will provide insight about the behavior of volatility in uncertain market and the impact of macroeconomic variables on volatility so that decision makers can take appropriate measures for investments and risk mitigation. Moreover this study will provide evidence about impact of value at risk and downside risk on determining the returns and volatility. Further this study will propose a nonlinear volatility based asset pricing models that will help in optimal decision making in areas of capital investment, financing, merger and acquisition and equity valuation.

GARCH model is linked here with the macroeconomic variables in our study to capture the effect of risk not by the variance series but also using the standard deviation of the series for mean and variance equations because the GARCH models allows us to add explanatory variables in the specification of the conditional variance equation that have ability to explain the variance though macroeconomic explanatory variables. The ultimate answer to such dilemma may assist to redesign and reframe the models of assets pricing as a better and exact solutions to the practical issues of optimal portfolio selection process, and also to provide assistance to watch and administer financial risks in an efficient way.

Therefore we will also take into consideration VaR and Semi-variance as risk measures for this GARCH extension approach as well. Such kind of results will provide more purposeful and useful answer about systematic financial sector risk to financial analysts, macroeconomists, central bankers, and big market players to get a wider range of consideration of latent macroeconomic determinants. There is no proper empirical evidence available yet in the existing literature that may assist an individual investor, institutional investors, banker and regulators to answer this question in a justified manner.

2. Literature Review

In reality, only a small amount of studies are accessible that report in the support of the suggestions for using the macroeconomic data for empirical research in finance in relation to stock return volatility modeling as Pierdzioch, Döpke and Hartmann (2008) explored the linkage between stock market volatility and macroeconomic factors but in a limited way but not focused on relation of interrelated volatilities and their predicting power of returns. Study concluded that stock market volatility likely to increase in the phase of downturns in business cycle Schwert (1989), Hamilton and Lin (1996) and Errunza and Hogan (1998). It is very interesting for investors that such kind of results put the questions that whether macroeconomic forces capture business cycle fluctuations and how these forces help to predict stock market volatility or not.

Engle and Rangel (2008) used macroeconomic factors to found volatility their study concluded that inflation, GDP growth, and short term interest rate are significant expounding variables that cause to an increase in the volatility. They concluded that inflation and growth of output are significant positive determinants of volatility.
Kulp-Tag (2008) visualized association between return volatility with volume and interest rates as impulse variables. The study explored that there exist concept of asymmetry in returns and volatility and the risk-return-information relationship is investigated through Standard and Poor 500 index based upon daily observations. The study concluded that interest rate is not an important information parameter for modeling the volatility and asymmetry in mean is important to model the conditional mean and variance. Here we can also hypothesis that if the return of KSE follows asymmetric patterns in the variance and mean then modeling may enhance the significance of the study under consideration.

On the other hand, Choo, Lee & Ung (2011) tested volatility of macroeconomic variables and performance of GARCH models. They concluded that the volatility of Japanese stock market is not affected by macroeconomic variables.

Engle Ghysels and Sohn (2013) studied the relationship between macroeconomic forces and stock market volatility by applying latest set of component models that differentiate short-run movements from secular actions. They formulated industrial production growth and inflation to drive the long-term component of model. Hence, it is concluded that adding economic factors into volatility models performs well in terms of long run predictions. Further they found that at every day level, industrial production growth and inflation take into consideration between ten percent and thirty five percent of one day forward volatility projection. Consequently, the study inferred that macroeconomic fundamentals play an important role even for short time horizons in capturing the volatility. It is concluded that the macroeconomic forces have the ability to capture the volatility in long run as well as in short run dynamics.

Sangmi and Hassan (2013) evaluated the macroeconomic variables impact on the stock price behavior and volatility of the Indian equity market. Their study concluded that there exist a significant relationship between equity market fluctuations and macroeconomic variables of inflation, interest rate, exchange rate, gold price, money supply and industrial production.

Attari & Safdar (2013) used EGARCH model to generate volatility from KSE return series and identified GDP, Inflation and interest rate as the key determinants of volatility in Pakistan.

Omorokunwa and Ikponmwosa (2014) examined the relationship between volatility of the stock market and macroeconomic variables such as GDP, exchange rate, interest rate and inflation. Empirical evidence is taken for the period of 1980 to 2011 by applying GARCH model. GARCH model capture the non-linear effects because volatility influence return in a non-linear fashion. It is the point that we may too hypothesize that the same behavior of volatility is reflected by KSE returns. They concluded that price behavior in Nigeria is volatile and the historical information has impact on stock market volatility in Nigerian equity market. Hence they concluded that exchange rate and interest rate have effect on stock price volatility in a weak manner and inflation is the major determinant in Nigerian stock price volatility. They suggested that inflation element should be taken into consideration in the proper design of targeted monetary policy by taking into the stock market perception of policies. In finance arbitrage pricing theory guides the relationship between macroeconomic variables and stock return.
d’Addona and Giannikos (2014) modeled asset pricing with business cycles in regimes switching in mean and variance equation. They identified model has predictability power and reports significant results. Further they realized and identified the modeling of macroeconomic risk in such kind of models.

It is evidence that Macroeconomic variables are significant information parameter for modeling the volatility and this hypothesis can be established for asset pricing in the emerging economies.

Herskovic et al. (2016) identified that idiosyncratic volatility leads a strong factor structure for pricing the common factors in idiosyncratic volatility for shocks. Lowest idiosyncratic volatility beta (Systemic Risk) has greater earning capacity than the highest idiosyncratic volatility beta. Therefore this particular element of idiosyncratic volatility assists to express the anomalies of asset pricing modeling as well. Epstein and Ji (2013) volatility and drift is modeled with a utility approach in a continuous time frame of reference and extension is made in asset pricing theory with arbitrage free rule, based upon arguments of hedging approach and sharp predictions can be attained by assuming preference maximization and equilibrium.

Demir, Fung, and Lu (2016) elaborated the performance of CAPM under a general equilibrium model, can be enhanced significantly by applying conditional consumption and market return volatilities as modeling factors. Indian market is tested through portfolios selected by size and book-to-market equity ratio point of view. Conditional volatility has very low effect on companies having large capitalization than small-growth and small-value based firms.

Kim and Kim (2016) modeled asset pricing and found strong evidence of Inter-linkages among the volatilities of 6 equity markets of United States and rejected the null hypothesis of constant volatility for the capital asset pricing model in the period of financial crises.

Brooks and Persand (2003), Yu (2006); McMillan and Speight (2007) used VaR techniques in the computation of stock return volatility in the Asian emerging markets. They identified that VaR is significant parameter for volatility modeling. However, there seem a lot of gap in existing literature with respect to VaR measurement in various equity markets.

Thupayagale (2010) analyzed the prediction performance by using GARCH model in context with Value-at-Risk estimation by using stock return data. The results reveals that models with asymmetric effects and having long memory are important in considering the provision of improved VaR estimates and can escape from losses in trade. Moreover the results indicate that it can be used to forecast for out-of-sample. It is an important parameter in the computation of Value-at-Risk for derivation of exact asset-return volatility estimations. It is inferred from the study that Value at Risk is significant information parameter for modeling the volatility. It may hypothesize that the same behavior prevails in the equity market dynamics of KSE. Volatility and asset pricing remained always a hot cake in financial modeling in various context and testified volatility in the domain of various risk anomalies and firm factors as Grootveld and Hallerbach (1999) indicated that semi-variance is same like to variance but it considers only values below the average value. This element refines the problems of asymmetry and known as downside risk. This element can be used to eliminate the probability of loss.
for the portfolio. Moreover this approach considers the element of lower partial moment that can be tested for empirical financial time series. It is inferred from the study that downside risk is significant information parameter for modeling the volatility.

2.1 Hypotheses of the Study

- **H₁**: Macroeconomic variables are significant information parameter for modeling the volatility.
- **H₂**: The return of KSE follows asymmetric patterns not only in the variance but also in the mean whereas volatility influence return in a non-linear fashion.
- **H₃**: Value at risk is significant information parameter for modeling the volatility and asset pricing for KSE returns and there exist a significant positive relationship between risk and return.
- **H₄**: Downside risk is significant information parameter for modeling the volatility and has significant positive impact on return.

3. Data and Methodology

3.1 Volatility and Return

This study will explore asset pricing on the basis of volatility. The process is explained with GARCH (p,q) and GARCH-in-Mean model that permits the conditional mean to depend upon its own conditional variance. If the risk is captured through volatility or by the conditional variance then the conditional variance may enter the conditional mean of $X_t$. Data for Macroeconomic modeling is taken for the period Jan 2000 to Dec 2015 on monthly basis. However for VaR and Semi-variance modeling we used daily data for the period of Jan 2000 to Dec 2015. Stock market returns are computed by using the following equation.

$$S_{rt} = ln\left(\frac{P_t}{P_{t-1}}\right) \tag{1}$$

$S_{rt}$ = Stock Returns

$P_t$ = Closing Price of Stock indices at time $t$

$P_{t-1}$ = Closing Price of Stock Indices at 1 time before.

3.1.1 Model 1: Return, Volatility and Macroeconomic Factors

The macroeconomics variable includes CPI, Term Structure of interest rate, industrial production and oil prices. The role of macroeconomic variable in determining volatility is modeled as under.

$$X_t = a_0 + \beta X_{t-1} + \gamma \sigma_t^2 + \pi_1 (Inflation)_t + \pi_2 (Interest rate change)_t$$
$$+ \pi_3 (Growth rate in real sector)_t + \pi_4 (Oil Price Change)_t$$
$$+ \mu_t \tag{2}$$

Whereas $X_t$ is return for $t$ periods and $a_0$ is constant and $\beta, \gamma$ and $\pi$ are slopes and coefficient. Whereas $X_t$ dependent variable $\sigma^2_t$ is variance and $\mu_t$.

$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i h_{t-i} + \sum_{j=1}^{q} \gamma_j \mu_{t-j} + \sum_{k=1}^{m} \mu_k M_k \tag{3}$$
Where $M_k$ is a set of macroeconomic explanatory variables that might help to explain the variance.

3.1.2 Model 2: Returns, Volatility, and Value at Risk

This study explains the dynamics of asset pricing and volatility in the presence of value at risk. The econometric model for said phenomena is provided below.

$$X_t = a_0 + \beta X_{t-1} + \gamma (\text{Value at risk})_t + \mu_t$$

Whereas $X_t$ is return for t periods and $a_0$ is constant and $\beta$, $\gamma$ and $\omega$ are slopes and coefficient. Whereas $X_t$, dependent variable and $\mu_t$ is error term.

$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i h_{t-i} + \sum_{j=1}^{q} \gamma_j \mu_{t-j}^2$$

Value at Risk (VaR) is a widely applied risk measure of the risk of loss on a specific portfolio of financial assets. For a given portfolio, probability and time horizon. Value at risk (VaR) measures the worst expected loss under normal market conditions for a specific time interval at a given confidence level. Value at risk answer to the question that how much can I lose with $x\%$ probability over a pre-set time horizon Jorion (1996).

3.1.3 Model 3: Returns, Volatility, Semi-variance

Above stated model are related to total risk as a measure of risk. The total risk is captured through standard deviation that demonstrates above and below mean value. Investor appreciates above mean market risk but concerned about downside risk deviation. So the downside risk is captured by using the following relationship.

$$X_t = a_0 + \beta X_{t-1} + \gamma (\text{Semi-variance})_t + \mu_t$$

Whereas $X_t$ is return for t periods and $a_0$ is constant and $\beta$, and $\gamma$ are slopes and coefficient. Whereas $X_t$, dependent variable $h_t$ is variance and $\mu_t$.

$$h_t = \gamma_0 + \sum_{i=1}^{p} \delta_i h_{t-i} + \sum_{j=1}^{q} \gamma_j \mu_{t-j}^2$$

Semi-variance is a measure of the dispersion of all observations that fall below the average or target value of a particular data set. The method for semi-variance computations is as follows:

$$\text{semivariance} = \frac{1}{n} \sum_{r_i < \text{average}} (\text{Average} - r_i)$$

Where: $n$ = Total number of observations below the mean, $r_i$ is observed value and average or target value of the data set. It is a useful tool in portfolio or assets analysis, semi-variance provides a measure for downside risk. Whereas standard deviation and variance are the measures of volatility but semi-variance only looks at the negative fluctuations of an asset. For risk averse investors, solving for optimal portfolio allocations by minimizing semi-variance would limit the likelihood of a large loss.
4. Results and Discussion

4.1 Econometric Modeling for KSE

Table 1: Estimates of GARCH in Mean (1, 1) Model 1: Return, Volatility and Macroeconomic Model for KSE: Impact of Macroeconomic Variables on Return

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Parameters</th>
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<td>Mean Equation</td>
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<tr>
<td></td>
<td>p-value</td>
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<tr>
<td></td>
<td>β</td>
<td>-0.00065</td>
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<tr>
<td></td>
<td>p-value</td>
<td>0.9949</td>
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<tr>
<td></td>
<td>γ</td>
<td>-0.2415</td>
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<tr>
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<td>p-value</td>
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<tr>
<td></td>
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<td>p-value</td>
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<tr>
<td></td>
<td>π₃</td>
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<tr>
<td></td>
<td>p-value</td>
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<td></td>
<td>π₄</td>
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<td></td>
<td>p-value</td>
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<td>Δ</td>
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<td>p-value</td>
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<td>γ₁</td>
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<tr>
<td></td>
<td>SIC- Statistics</td>
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<tr>
<td></td>
<td>Log- Likelihood</td>
<td>219.5998</td>
</tr>
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</table>

Table 1 indicates that GARCH-in-mean model is extended with the macroeconomic variables in the variance equation for KSE. The conditional mean is not significant. So far as macroeconomic variables are concerned, change in interest rate has significant negative effect on return and increase in change in interest rate will decrease returns of stocks. The change in oil prices has also significant positive impact on KSE returns. The performance of the model is checked on the basis of AIC, SIC, and Log Likelihood values.
Table 2 indicates the impact of macroeconomic variables on volatility of the market has also been exercised. The results indicate that interest rate is significant positively related to volatility. In high periods of interest rate, volatility is on high side. Therefore in the period of rising prices volatility is lower it may be due to anchoring. However oil prices change has negative significant impact on volatility.
Figure 1: Trend of KSE-100 Index

Figure 2: Return Behavior of KSE

Figure 3: Histogram of KSE Returns

Series: Standardized Residuals
Sample 1/06/2000 12/31/2015
Observations 3937
Mean -0.034590
Median -0.017374
Maximum 3.766339
Minimum -6.735602
Std. Dev. 0.999702
Skewness -0.595070
Kurtosis 5.971998
Jarque-Bera 1681.296
Probability 0.000000
Figure 4: Trend of Macro Economic Variables and Stock Returns

Figure 5: Conditional Volatility of KSE-100

Figure 6: Conditional Risk of KSE
Value at Risk

Figure 7: Value at Risk

Factor Analysis For Macro Economic Variables and Stock Returns

Figure 8: Factor Analysis For Macro Economic Variables and Stock Returns
The Figure 1 indicates the trend of KSE-100 index based upon the prices and indicates a sharp rise after year 2008 to onward. Whereas, Figure 2 indicates the behavior of returns in KSE and this return density indicate the above and below fluctuations from zero level. Figure 3 represents the histogram of KSE returns density. Returns shows a leptokurtic behavior and the data is negatively skewed with -0.55 values as given by the stated statistics. The overall mean return shows negative value of -0.034 with a 0.999 standard deviation. However Jarque-Bera value indicates the normal behavior of the data as p<0.0000001.

Figure 4 indicates the trend of macroeconomic variables and stock returns in a log linear fashion. Figure 5 shows the conditional volatility behavior of stock returns created by GARCH based model and indicates the asymmetric patterns in the volatility of KSE. It is clearly visualized that high period volatility follows high volatility and in the period of low volatility follows low volatility and shows the persistency in the volatility level. Figure 6 shows the conditional standard deviation (Conditional Risk) for KSE and shows the asymmetric patterns and leverage effect with persistency in volatility.

Figure 7 indicates the Value at Risk behavior for KSE and Figure 8 indicates the factor analysis for macroeconomic and stock returns.

Table 3: Estimates of GARCH (1,1)/Model 3: Return, Volatility and Value at Risk for KSE

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<td>Log likelihood</td>
<td>29753.16</td>
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Table 3 indicates the relationship of return and the value at risk. GARCH Model is extended with the Value at Risk in mean equation. The results indicate that the γ is negatively related to return significantly. It is inferred that VaR is significantly negatively
related to the returns of KSE market. ARCH term is significant at 95% confidence interval indicating that past price behavior influence current volatility in the market. The GARCH term is significant at 95% confidence interval which reports the presence of persistence in the volatility. It indicates that the value at risk is negative and has effect on the price behavior.

Table 4: Estimates of GARCH (1,1) Model 4: Return, Volatility and Semi-Variance for KSE

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<td>Variance Equation</td>
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<td></td>
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<td>Log-likelihood</td>
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Table 4 indicates the relationship of return, and the Semi-variance. GARCH Model is extended with the Semi-variance. Semi-variance is downside risk and added into variance equation. Here semi-variance is significant and have negative impact. The results indicate that ARCH term and GARCH term are significant at p<0.00001. Here all the variables for variance equation are statistically significant.
Table 5: Diagnostic Test

<table>
<thead>
<tr>
<th>Table</th>
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<td>SIC- Statistics</td>
<td>-2.03542</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Log Likelihood</td>
<td>219.5998</td>
</tr>
<tr>
<td>2</td>
<td>b.</td>
<td>AIC- Statistics</td>
<td>-6.16624</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SIC- Statistics</td>
<td>-6.14921</td>
</tr>
<tr>
<td></td>
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<td>Log Likelihood</td>
<td>7323.241</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>AIC- Statistics</td>
<td>-15.1116</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SIC- Statistics</td>
<td>-15.102</td>
</tr>
<tr>
<td></td>
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<td>Log Likelihood</td>
<td>29753.16</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td>AIC- Statistics</td>
<td>-5.74417</td>
</tr>
<tr>
<td></td>
<td></td>
<td>SIC- Statistics</td>
<td>-5.7265</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Log Likelihood</td>
<td>5411.259</td>
</tr>
</tbody>
</table>

Table 5 indicates the summary of diagnostic test for KSE. AIC, SIC and Log likelihood values are used to select the model that may best model the conditional mean and conditional variance for these equity markets in a best way. First of all for model 1(a), KSE, has lower values for AIC, SIC, Log Likelihood and it indicates that the conditional mean can be modeled for this economy for asset pricing in a best way along with the extension of macroeconomic variables in GARCH in Mean Model. For Model 1(b) AIC, SIC and Log likelihood values are used to select the model that may best model the conditional variance for these equity markets in a best way and it is ranked at third level for KSE and it indicates that the conditional variance can be modeled in these economy for asset pricing in a best way along with the extension GARCH in mean model with macroeconomic variables in the variance equation. However the performance of the model 1(b) cannot be rejected at all due to its ranking level. Model 2 ensures that KSE can be modeled along with VaR to explain the risk return relationship in this economy but the model is ranked at fourth level based upon AIC, SIC and Log Likelihood Criteria. Finally Model 3 is performing best for KSE as well because it is ranked at two on the basis of AIC, SIC and Log Likelihood Criteria.

5. Conclusion

After thorough analysis and interpretations it is summarized and concluded that study meets the objectives and identifies that the conditional volatility can be modeled in an uncertain market with specific information parameters. For this identification we extended GARCH in mean model with macroeconomic variables for KSE in first instance. Impact of macroeconomic variable on return and volatility is tested in mean and variance equation simultaneously. GARCH in mean model is extended with the macroeconomic variables in the variance equation for KSE and the change in interest rate has significant negative effect on return and increase in change in interest rate will
decrease returns of stocks. The change in oil prices has also significant positive impact on KSE returns. Further the impact of macroeconomic variables on volatility of KSE market has also been exercised. The results indicate that interest rate is significantly positively related to volatility. In high periods of change in interest rate, volatility is on high side. Therefore in the period of rising prices volatility is lower it may be due to anchoring. However oil prices change has also negative significant impact on volatility. It is concluded that macroeconomic variables are significant parameters for explaining the returns of stock as well as volatility in these markets. Attari & Safdar (2013), Omorokunwa and Ikponmwosa (2014) Sangmi and Hassan (2013) also identified that interest rate is significant volatility determinant in the same perspective and support our study argument. However Kulp-Tag (2008) results indicate that interest rate is not significant determinant of volatility. However our results does not coincide with the studies of Engle and Rangel (2008), Choo, Lee & Ung (2011) Engle Ghysels and Sohn (2013). Hence it is justified that Macroeconomic variables are significant information parameter for modeling the volatility in this economy.

Further the relationship of return and the value at risk is explored for all the equity markets. GARCH (1,1) Model is extended with the Value at Risk in mean equation. The results indicate that the $\gamma$ is negatively related to return significantly. It is inferred that VaR is significantly negatively related to the returns of KSE market. AIC, SIC and Log likelihood values are used to select the model that may best model the conditional mean and conditional variance for these equity markets in a best way. The studies of Thupayagale (2010), Brooks and Persand (2003), Yu (2006); McMillan and Speight (2007) also support our argument.

Further GARCH (1,1) Model is extended with the Semi-variance for KSE. Semi-variance is downside risk and added into mean equation. Here semi-variance is significant which indicates that downside risk has negative impact. Here all the variables for variance equation are statistically significant. It is concluded that semi-variance is one of the risk determinant that explain the volatility behavior in the market. This element is also supported by the arguments of Hallerbach (1999).

Above all it is concluded that there exist a significant positive relationship between risk and return in the Karachi Stock Market. The emerging stock market follows asymmetric patterns not only in the variance, but also in the mean. No doubt there exists asymmetry in the variance for this emerging market and negative reactions increases volatility more than positive reactions in the KSE. Study further concludes that small positive shocks have a larger impact on the conditional volatility than small negative shocks; however when the reactions are greater in size, then the effect on volatility is in opposite direction. This element elaborates that large positive shocks contributes to a smaller increase in volatility rather than large shock is negative. The returns of KSE follows asymmetric pattern in mean in which positive returns are followed by more positive returns but negative returns revert to positive returns faster than positive reverts to the negative returns. It provides that volatility influences returns in a non-linear fashion. It is finally concluded that volatility plays a significant role in pricing of financial assets in emerging economies.

In almost previous work the predictability have mostly dealt with the behavior of stock returns in simple ARCH/GARCH models and the literature lacks issues in asset pricing regarding to capture the non-linear behavior of stock returns and volatilities through
Macroeconomic, VaR and Downside risk proxies in context to Pakistan. So this study contributes the assessment of behavior of risk and return in the Equity market of Pakistan for the prime interest of the investors. This study contributes for decision makers to take appropriate measures for mitigating risk for their investments. In short now this study proposed a nonlinear volatility based asset pricing model that will help in optimal decision making in areas of capital investment, financing, merger and acquisition and equity valuation in the money and capital of market of Pakistan.

Therefore it is advised to the investors that they may use investment strategies by analyzing recent and historical news, information shocks and can forecast the future market movements based upon these models and can use this information for selecting optimal portfolio for efficient risk management to harvest stream of benefits in such economy.

The first limitation of the study included that we used only historical returns of KSE-100 only and only four macroeconomic variables are taken. Secondly GARCH model is used only to reveal the phenomena of volatility and heteroskedastic element and the extension approaches reflects the limitations of the normal distribution of data assumption to assess the historical volatility of market risk. The third limitation indicates that we used only VaR and Downside risk as risk proxies to model the volatility of stock return.

This study provides a base for future directions on the grounds that macroeconomic information parameters and risk proxies can be extended with other models of GARCH family like EGARCH, GJR-GARCH, VS-GARCH, QARCH for asset pricing and volatility predictions.

REFERENCES


