An Overview on Method of Cyclic Shifts for the Construction of Experimental Designs Useful in Business and Commerce

Rashid Ahmed (Corresponding author)
Department of Statistics, The Islamia University of Bahawalpur, Pakistan
Email: rashid701@hotmail.com

M. H. Tahir
Department of Statistics, The Islamia University of Bahawalpur, Pakistan
Email: mtahir.stat@gmail.com

Muhammad Rajab
Department of Statistics, The Islamia University of Bahawalpur, Pakistan
Email: rjabmalik@yahoo.com

Muhammad Daniyal
Department of Statistics, The Islamia University of Bahawalpur, Pakistan
Email: muhammad.daniyal@iub.edu.pk

Abstract
The competitive nature of business today requires the knowledge to be able to manipulate process levels in a predictable fashion and to reduce process variation (quality control). This goal can be achieved through the use of experimentation to identify and confirm the effects of the process variables. Therefore, design of experiments should be an integral part of process management. Experimental designs such as treatment balanced, neighbor balanced and balanced repeated measurements designs, have been extensively used in different fields, especially in business and commerce. Different methods have been used for their construction but method of cyclic shifts is the easiest one. This paper provides the overview, how this method has been used for the construction of the useful designs.

Keywords: cyclic shifts; experimental designs; competitive nature of business; polygonal designs; process management; quality control.

1. Introduction
Method of cyclic shifts (MOCs) has been widely used to construct different designs. This method was developed by Iqbal (1991). In this method, we can study some standard characteristics and properties of the specific design only through the set(s) of shifts. This method is quite simple to construct several types of important designs. Some of these are balanced incomplete block designs (BIBDs), partially BIBDs, polygonal designs (PDs), neighbor balanced designs (NBDs) and repeated measurements designs (RMDs). In this
Method of Cyclic Shifts for the Construction of Experimental Designs

article, an overview is provided that how the MOCS is used to construct several types of important designs. This method consists of Rule I & Rule II.

According to Sanders et al. (2002), the competitive nature of business today requires the knowledge to be able to manipulate process levels in a predictable fashion and to reduce process variation. The need of this knowledge (the continuous improvement of processes) must be quicker than ever before. This goal requires the ongoing use of experimentation to identify and confirm the effects of the process variables. Therefore, designed experiments should be viewed as an integral part of process management. Whereas the statistical design of experiments (DOE) is a valuable tool for rapidly accumulating this process knowledge. In industrial experimentation, blocks are frequently a period of time when extraneous variables (variables not explicitly manipulated in the experiment) can reasonably be expected to remain constant while an experiment takes place. Periods of time (blocks) for subsequent experiments are selected so that while variables not explicitly manipulated can reasonably be expected to remain constant during the execution of a DOE, some might have changed between experiments (blocks). Polygonal designs are useful for survey and marketing.

2. Construction of BIBDs and PBIBDs

BIBDs and PBIBDs have the most significant role in experimental designs to compare each pair of treatments with equal or almost equal precision. Such experimental designs provide the assurance that treatments can be compared with same precision. Using method of cyclic shifts, Yasmin et al. (2015) and Jamil et al. (2017) presented BIBD and PBIBD respectively.

2.1 How to Obtain a BIBD and PBIBD Using Rule I.

MOCS is described in this Section only for BIBDs and PBIBDs.

Let \( S_j = [q_{j1}, q_{j2}, ..., q_{j(k-1)}] \), \( 1 \leq q_{ji} \leq v-1 \). A design will be BIBD if each of 1, 2, ..., \( v-1 \) appears \( \lambda \) times in \( S_j^* \). \( S_j^* = [q_{j1}, q_{j2}, ..., q_{j(k-1)}], (q_{j1}+q_{j2}), (q_{j2}+q_{j3}), ..., (q_{j(k-2)}+q_{j(k-1)}), (q_{j1}+q_{j2}+q_{j3}), ..., (q_{j1}+q_{j2}+...+q_{j(k-1)}) \), and complement of each element], here \( v-q_i \) is complement of \( q_i \). It will be PBIBD (two associate) if \( \lambda \) has two values \( \lambda_2 = \lambda_1 + 1 \).

Example 2.1: \( S = [3, 1, 2, 2, 1, 1, 1] \) provide BIBD for \( v = 15 \), \( k = 8 \) in 15 blocks through MOCS (Rule I).
Here
\[ S' = [3, 1, 2, 2, 1, 1, 4, 3, 4, 3, 2, 2, 6, 5, 4, 3, 8, 6, 6, 5, 9, 7, 7, 10, 8, 11, 12, 14, 13, 14, 13, 14, 14, 11, 12, 11, 12, 13, 9, 10, 11, 12, 7, 9, 9, 10, 6, 8, 8, 5, 9, 7, 7, 10, 8, 11, 12, 14, 13, 13, 14, 14, 14, 11, 12, 11, 12, 13, 9, 10, 11, 12, 7, 9, 9, 10, 6, 8, 8, 5, 7, 4]. \] Here each element from 1, 2, 3, \ldots, 14 appears 4 times. Hence it is BIBD with \( \lambda = 4 \).

<table>
<thead>
<tr>
<th>Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
<tr>
<td>6</td>
</tr>
<tr>
<td>8</td>
</tr>
<tr>
<td>9</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>11</td>
</tr>
</tbody>
</table>

2.2 How to Obtain a BIBD and PBIBD Using Rule II

Let \( S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}]^t, 1 \leq q_j \leq v-2 \). A design will be BIBD if each of 1, 2, \ldots, \( v-2 \) appears \( \lambda \) times in \( S_j^* \). Where \( S_j^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}, (q_{j1}+q_{j2}), (q_{j2}+q_{j3}), \ldots, (q_{j(k-4)}+q_{j(k-3)}+q_{j(k-2)}), (q_{j1}+q_{j2}+q_{j3}), (q_{j2}+q_{j3}+q_{j4}), \ldots, (q_{j(k-4)}+q_{j(k-3)}+q_{j(k-2)}), \ldots, (q_{j1}+q_{j2}+\ldots+q_{j(k-2)})] \), here, \( v-1-q_j \) is complement of \( q_j \). If \( \lambda \) has two values \( \lambda_2 = \lambda_1 + 1 \) then it is PBIB design with 2-association scheme.

Example 2.2: BIBD is constructed from the sets of shifts \([1, 2, 3]\) and \([1, 4]\) for \( v = 8, k = 4 \) in 14 blocks through Rule II. Here \( S' = [1, 2, 3, 3, 5, 6, 1, 4, 5, 6, 5, 4, 2, 1, 6, 3, 2] \) Here each element from 1, 2, 3, \ldots, 6 appears 3 times. Hence it is BIBD with \( \lambda = 3 \).

<table>
<thead>
<tr>
<th>Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
</tr>
<tr>
<td>0</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>6</td>
</tr>
</tbody>
</table>
3. Construction of Polygonal Designs

PDs are important in survey sampling. Using MOCS, Tahir et al. (2011) constructed PDs for $k = 3$ with $\lambda = 1, 2, 3, 4, 6, 12$ and $\alpha = 2$, Intizar et al. (2016) constructed PDs in blocks of two different sizes 4 and 2. Following is the procedure to obtain PDs.

3.1 How to Obtain Polygonal Designs Using Rule I

Let $S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-1)}]$, where $1 \leq q_{ji} \leq v-1$.

- If each of 2, ..., $v-2$ appears $\lambda$ times in $S^*$ except 1 and $v-1$ which do not appear then this design is CPD with $\alpha = 1$.
- If each of 3, ..., $v-3$ appears $\lambda$ times in $S^*$ except 1, 2, $v-1$ and $v-2$ which do not appear then this design is CPD with $\alpha = 2$.
- If each of $\alpha+1$, ..., $v-(\alpha+1)$ appears $\lambda$ times in $S^*$ except 1, 2, ..., $\alpha$, $v-1$, $v-2$, ..., $v-\alpha$ which do not appear then this design is CPD with joint distance $\alpha$.

Here, $S^*$ contains (i) each shift of $S$, (ii) sum (mod $v$) of each of two, three, ..., $k-1$ consecutive shifts, and (iii) complement of each element in (i) & (ii). Here, $v-q_i$ is complement of $q_i$.

For a circular polygonal design with $\alpha = 1$, the concurrence matrix is:

$$NN' = \begin{pmatrix} r & 0 & \lambda & \ldots & \lambda & 0 \\ 0 & r & 0 & \ldots & \lambda & \lambda \\ \lambda & 0 & r & \ldots & \lambda & \lambda \\ \lambda & \lambda & 0 & r & \ldots & \lambda \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \ldots & r & 0 \\ 0 & \lambda & \lambda & \ldots & 0 & r \end{pmatrix}$$

For a circular polygonal design with joint distance $\alpha = 2$ the concurrence matrix is

$$NN' = \begin{pmatrix} r & 0 & 0 & \lambda & \ldots & \lambda & 0 & 0 \\ 0 & r & 0 & 0 & \ldots & \lambda & \lambda & 0 \\ 0 & 0 & r & 0 & \ldots & \lambda & \lambda & \lambda \\ \lambda & 0 & 0 & r & \ldots & \lambda & \lambda & \lambda \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\ \lambda & \lambda & \lambda & \ldots & r & 0 & 0 & 0 \\ 0 & \lambda & \lambda & \lambda & \ldots & 0 & r & 0 \\ 0 & 0 & \lambda & \lambda & \lambda & \ldots & 0 & 0 & r \end{pmatrix}$$

Example 3.1: A circular polygonal design for $v = 11, k_1 = 3, k_2 = 2, \lambda = 1$ & $\alpha = 1$ is constructed using $[2, 3]+[4]$. The required design is:
3.2 How to Obtain Polygonal Designs Using Rule II

Let \( S_j = [q_{j1}, q_{j2}, ..., q_{j|k-2|}] \), where \( 1 \leq q_{ji} \leq \nu-1 \).

- If each of \( 2, ..., \nu-3 \) appears \( \lambda \) times in \( S^* \) except 1 and \( \nu-2 \) which do not appear then this design is CPD with \( \alpha = 1 \).
- If each of \( 3, ..., \nu-4 \) appears \( \lambda \) times in \( S^* \) except 1, 2, \( \nu-2 \) and \( \nu-3 \) which do not appear then this design is CPD with \( \alpha = 2 \).
- If each of \( \alpha+1, ..., \nu-1-(\alpha+1) \) appears \( \lambda \) times in \( S^* \) except 1, 2, ..., \( \alpha \), \( \nu-2, ..., \nu-1-\alpha \) which do not appear then this design is CPD with joint distance \( \alpha \).

Example 3.2: \([2,3]+[4]\) provide PD for \( \nu = 8, k = 3, \lambda = 2 \) and \( \alpha = 1 \).

Here \( S^* = [2,3,5,4,5,4,2,3] \) Here each element from 2, 3, ..., 5 appears exactly twice. Hence it is polygonal design with \( \lambda = 2 \) and \( \alpha = 1 \).

4. Construction of Balanced and Strongly Balanced RMDs

RMDs have application in many branches such as business, commerce, agriculture, food science, animal husbandry, biology, education, psychology, and pharmacology. In the following literature, MOCS is used to construct RMDs.
Method of Cyclic Shifts for the Construction of Experimental Designs

Iqbal and Jones (1994), Iqbal and Tahir (2009), Iqbal et al. (2010), Bashir et al. (2018), Rajab et al. (2018), Rasheed et al. (2018), Ahmed et al. (2018), Khan et al. (2019), Daniyal et al. (2019), Rasheed et al. (2019), and Ahmed et al. (2019) constructed CBRMDs & CSBRMDs in periods of equal and unequal sizes. Jabeen et al. (2019), Nazeer et al. (2019), Jabeen et al. (2019) constructed minimal CSPBRMDs in periods of equal and two different sizes. Hussain et al. (2020) constructed CWBRMDs in two different periods sizes. Abdullah et al. (2019) constructed such designs in non-circular periods. Following is the to obtain the BRMDs.

4.1 How to Obtain BRMDs in Circular Periods using Rule I

Let \( S = [q_1, q_2, ..., q_{v-1}] \), where \( 1 \leq q_i \leq v-1 \). If each of \( 1, 2, ..., v-1 \) appears \( \lambda' \) in \( S^* \) then it be CBRMD in periods of size \( p \), where \( S^* = [q_1, q_2, ..., q_{v-1}, v - (q_1 + q_2 + ... + q_{v-1}) \mod v] \). Design will be non-binary if sum of any two, three, ..., or \( (p-1) \) consecutive elements of \( S \) is 0 (mod \( v \)), otherwise binary.

Example 4.1: Set of shifts \([1,2,8]+[3,4,6]\) produce non-binary CBRMD for \( v = 9 \) and \( p = 4 \).

Here \( S^* = [2,1,8,7,3,4,6,5] \) Here each element from 1,2,3,...,8 appears once. Therefore, it is minimal CBRMDs with \( \lambda' = 1 \).

<table>
<thead>
<tr>
<th>Periods</th>
<th>Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>2</td>
<td>0 2 4 6 8</td>
</tr>
<tr>
<td>3</td>
<td>1 3 5 7 9</td>
</tr>
<tr>
<td>4</td>
<td>2 0 4 6 8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Periods</th>
<th>Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>1 2 3 4 5 6 7 8 9</td>
</tr>
<tr>
<td>11</td>
<td>1 3 5 7 9</td>
</tr>
<tr>
<td>12</td>
<td>2 4 6 8</td>
</tr>
<tr>
<td>13</td>
<td>0 2 4 6 8</td>
</tr>
<tr>
<td>14</td>
<td>1 3 5 7 9</td>
</tr>
<tr>
<td>15</td>
<td>0 2 4 6 8</td>
</tr>
<tr>
<td>16</td>
<td>1 3 5 7 9</td>
</tr>
<tr>
<td>17</td>
<td>0 2 4 6 8</td>
</tr>
<tr>
<td>18</td>
<td>1 3 5 7 9</td>
</tr>
</tbody>
</table>

4.2 How to Obtained BRMDs in Circular Blocks Using Rule II

Let \( S_1 = [q_{11}, q_{12}, ..., q_{p_1-1}] \) and \( S_2 = [q_{21}, q_{22}, ..., q_{p_2-1}] \), where \( 1 \leq q_i \leq v-2 \). If each of \( 1, 2, ..., v-2 \) appears \( \lambda' \) in \( S^* \) then it is CBRMD in periods of size \( p \), where \( S^* = [q_{11}, q_{12}, ..., q_{p_1-1}, (v-1) - (q_{11} + q_{12} + ... + q_{p_1-1}) \mod (v-1)], q_{21}, q_{22}, ..., q_{p_2-2} \).
Example 4.2: Sets of shifts [2,1,6]+[3,4] provide following non-binary CBRMD for \( v = 8 \), \( k = 4 \). Here \( S^* = [1,2,6,5,3,4] \) and each element from 1, 2, 3, …, 6 appears exactly once. Hence it is minimal CBRMDs with \( \lambda' = 1 \).


<table>
<thead>
<tr>
<th>Periods</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0(_2)</td>
<td>1(_3)</td>
<td>2(_4)</td>
<td>3(_5)</td>
<td>4(_6)</td>
<td>5(_7)</td>
<td>6(_1)</td>
<td>0(_7)</td>
<td>1(_8)</td>
<td>2(_9)</td>
<td>3(_10)</td>
<td>4(_11)</td>
<td>5(_12)</td>
<td>6(_13)</td>
</tr>
<tr>
<td>2</td>
<td>2(_6)</td>
<td>3(_1)</td>
<td>4(_2)</td>
<td>5(_3)</td>
<td>6(_4)</td>
<td>0(_5)</td>
<td>1(_6)</td>
<td>3(_0)</td>
<td>4(_1)</td>
<td>5(_2)</td>
<td>6(_3)</td>
<td>0(_4)</td>
<td>1(_5)</td>
<td>2(_6)</td>
</tr>
<tr>
<td>3</td>
<td>3(_2)</td>
<td>4(_3)</td>
<td>5(_4)</td>
<td>6(_5)</td>
<td>0(_6)</td>
<td>1(_0)</td>
<td>2(_1)</td>
<td>3(_2)</td>
<td>4(_3)</td>
<td>5(_4)</td>
<td>6(_5)</td>
<td>0(_6)</td>
<td>1(_0)</td>
<td>2(_1)</td>
</tr>
<tr>
<td>4</td>
<td>2(_3)</td>
<td>3(_4)</td>
<td>4(_5)</td>
<td>5(_6)</td>
<td>6(_0)</td>
<td>0(_1)</td>
<td>1(_2)</td>
<td>7(_7)</td>
<td>7(_1)</td>
<td>7(_2)</td>
<td>7(_3)</td>
<td>7(_4)</td>
<td>7(_5)</td>
<td>7(_6)</td>
</tr>
</tbody>
</table>

4.3 How to Obtain a CSBRMD Using Rule I

Let \( S_1 = [q_{11}, q_{12}, \ldots, q_{1(p-1)}] \), where \( 0 \leq q_{ij} \leq v-1 \). If each of 0, 1, 2, …, \( v-1 \) appears \( \lambda' \) in \( S^* \) then it is CSBRMDs in period of size \( p \), where \( S^* = [q_{11}, q_{12}, \ldots, q_{1(p-1)}, v-(q_{11} + q_{12} + \ldots + q_{1(p-1)}) \mod v] \). It will be minimal if \( \lambda' = 1 \).

Example 4.3: \([1,3,2,4,6,5]\) provides following CSBRMDs for \( v = 7 \) and \( p = 7 \). Here \( S' = [1,3,2,4,6,5,0] \) Here each element from 0,1,2,3,…,6 appears exactly once. Hence it is minimal CSBRMDs with \( \lambda' = 1 \) using rule I.

<table>
<thead>
<tr>
<th>Periods</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0(_0)</td>
<td>1(_1)</td>
<td>2(_2)</td>
<td>3(_3)</td>
<td>4(_4)</td>
<td>5(_5)</td>
<td>6(_6)</td>
</tr>
<tr>
<td>2</td>
<td>1(_0)</td>
<td>2(_1)</td>
<td>3(_2)</td>
<td>4(_3)</td>
<td>5(_4)</td>
<td>6(_5)</td>
<td>0(_6)</td>
</tr>
<tr>
<td>3</td>
<td>4(_1)</td>
<td>5(_2)</td>
<td>6(_3)</td>
<td>0(_4)</td>
<td>1(_5)</td>
<td>2(_6)</td>
<td>3(_0)</td>
</tr>
<tr>
<td>4</td>
<td>6(_0)</td>
<td>0(_5)</td>
<td>1(_6)</td>
<td>2(_0)</td>
<td>3(_1)</td>
<td>4(_2)</td>
<td>5(_3)</td>
</tr>
<tr>
<td>5</td>
<td>3(_6)</td>
<td>4(_0)</td>
<td>5(_1)</td>
<td>6(_2)</td>
<td>0(_3)</td>
<td>1(_4)</td>
<td>2(_5)</td>
</tr>
<tr>
<td>6</td>
<td>2(_3)</td>
<td>3(_4)</td>
<td>4(_5)</td>
<td>5(_6)</td>
<td>6(_0)</td>
<td>0(_1)</td>
<td>1(_2)</td>
</tr>
<tr>
<td>7</td>
<td>0(_2)</td>
<td>1(_3)</td>
<td>2(_4)</td>
<td>3(_5)</td>
<td>4(_6)</td>
<td>5(_0)</td>
<td>6(_1)</td>
</tr>
</tbody>
</table>

4.4 How to Obtain BRMDs in Linear Periods Using Rule I

Let \( S = [q_1, q_2, \ldots, q_{p-1}] \), where \( 1 \leq q_i \leq v-1 \). If each of 1, 2, …, \( v-1 \) appears \( \lambda' \) in \( S^* \) then it is BRMD in linear periods of size \( p \), where \( S^* = [q_1, q_2, \ldots, q_{p-1}] \).
Example 4.4: Sets of shifts \([2, 1, 3, 4]+[5, 6, 8, 7]\) provide following CBRMDs in linear periods for \(v = 9\) in \(p = 5\).

Here \(S^* = [2,1,3,4,5,6,8,7]\) and each element from 1, 2, 3, ..., 8 appears once, therefore, it is minimal BRMDs with \(\lambda' = 1\) in linear periods, using rule I.

\[
\begin{array}{cccccccccccccccc}
\text{Periods} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
2 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 \\
3 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 \\
4 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
5 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 \\
\end{array}
\]

4.5 How to Obtain BRMDs in Linear Periods Using Rule II

Let \(S_i = [q_i, q_{i+1}, \ldots, q_{i+p-1}]\) and \(S = [q_1, q_2, \ldots, q_{p-1}]\), where \(1 \leq q_0 \leq v-2\). If each of 1, 2, ..., \(v-2\) appears \(\lambda'\) in \(S^*\) then it is non-linear BRMD in periods of size \(p\), where \(S^* = [q_1, q_2, \ldots, q_{p-1}] \mod (v-1)\).

Example 4.5: Sets of shifts \([1,2,3,4]+[5,6,7]\) provide following CBRMDs for \(v = 9\) & \(p = 5\) in linear periods, using Rule II.

Here \(S^* = [1,2,3,4,5,6,7]\) and each element from 0, 1, 2, 3, ..., 7 appears exactly once. Hence it is minimal BRMDs in linear blocks with \(\lambda' = 1\), using Rule II.

\[
\begin{array}{cccccccccccccccc}
\text{Periods} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \\
1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\
2 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 \\
3 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 \\
4 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
5 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 1 \\
\end{array}
\]

4.6 How to Obtain SBRMD in Linear Periods Using Rule I

Let \(S = [q_1, q_2, \ldots, q_{p-1}]\), where \(0 \leq q_i \leq v-1\). If each of 1, 2, ..., \(v-1\) appears \(\lambda'\) in \(S^*\) then it is BRMD in linear periods of size \(p\), where \(S^* = [q_1, q_2, \ldots, q_{p-1}]\).

Example 4.6: Set of shifts \([1,2,3,0]+[4,5,6,7]\) produce SBRMD for \(v = 8\) & \(p = 5\) in linear periods.
Here $S^* = \{1,2,3,0,4,5,6,7\}$ Here each element from 0,1,2,3,…,7 appears exactly once. Hence it is minimal SBRMDs in linear blocks with $\lambda' = 1$, using Rule I.

<table>
<thead>
<tr>
<th>Periods</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>0</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

5. Construction of Neighbor Balanced Design

NBDs are used to control neighbor effects. NBDs constructed through MOCS up to 2011 can be found in Ahmed et al. (2011), Ahmed and Akhtar (2011), Ahmed and Akhtar (2012 a, b) Yasmin et al. (2013), Ahmed et al. (2013), Ahmed and Akhtar (2013), Yasmin et al. (2014), Ahmed et al. (2014), Ahmed and Akhtar (2015), Ahmed et al. (2016), Shahid et al. (2017), Ahmed et al. (2017), Khalid et al. (2018), and Shahid et al. (2019) constructed circular and non-circular NBDs. Following is the procedure to obtain NBDs.

5.1 How to Obtain NBDs in Linear Blocks Using Rule I

Let $S = [q_{11}, q_{12}, \ldots, q_{1(v-1)}]$, where $1 \leq q_{1i} \leq v-1$. If each of 1, 2, …, v-1 appears $\lambda'$ times in $S^*$ then design is NBD in linear blocks, where $S^* = [q_{11}, q_{12}, \ldots, q_{1(v-1)}] - vq_{11}, v-q_{12}, \ldots, v-q_{1(v-1)}$.

Example 5.1: Sets of shifts $[1,2,3]+[4,5,6]$ provide NBD for $v = 13$ in linear blocks of size 4 with $\lambda' = 1$.

Here $S^* = [1,2,3,4,5,6,12,11,10,9,8,7]$ and each element from 1, 2, 3, …, 12 appears exactly once. Hence it is minimal NBD in linear blocks using Rule I.
5.2 How to Obtain NBDs in Linear Blocks Using Rule II

Let \( S_j = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}] \), where \( 1 \leq q_{ji} \leq v-2 \). If each of 1, 2, \ldots, \( v-2 \) appears \( \lambda' \) times in \( S^* \), design is NBD in linear blocks, where \( S^* = [q_{j1}, q_{j2}, \ldots, q_{j(k-2)}, v-1-q_{j1}, v-1-q_{j2}, \ldots, v-1-q_{j(k-2)}] \).

Example 5.2: \([1,2]+[3]\) produce minimal NBD for \( v = 8 \) & \( k = 3 \).

Here \( S^* = [1,2,3,6,5,4] \) and each element from 1, 2, 3, \ldots, 6 appears exactly once. Hence it is minimal NBD in linear blocks using Rule II.

5.3 How to Obtain Nearest Neighbor Balanced Design Using Rule I

Rule I: Let \( S = [q_1, q_2, \ldots, q_k] \), where \( 1 \leq q_i \leq v-1 \). If each of 1, 2, \ldots, \( v-1 \) appears \( \lambda' \) in \( S^* \) then it is CNBD, where \( S^* = [q_1, q_2, \ldots, q_k, (q_1+q_2+\ldots+q_k) \mod v, v-(q_1), v-(q_2), \ldots, v-(q_k)] \).

Example 5.3: Set of shifts \([1,2,3,9,5]\) provides minimal NBD for \( v = 13 \) and \( k = 6 \) from.
Here \( S^* = [1,2,3,9,5,7,6,8,4,10,11,12] \) and each element from 1, 2, 3, ..., 12 appears once, therefore, it is minimal CNBDs, using Rule I.

<table>
<thead>
<tr>
<th>Blocks</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 1 2 3 4 5 6 7 8 9 10 11 12 0</td>
</tr>
<tr>
<td>1 2 3 4 5 6 7 8 9 10 11 12 0</td>
</tr>
<tr>
<td>3 4 5 6 7 8 9 10 11 12 0 1 2</td>
</tr>
<tr>
<td>6 7 8 9 10 11 12 0 1 2 3 4 5</td>
</tr>
<tr>
<td>2 3 4 5 6 7 8 9 10 11 12 0 1</td>
</tr>
<tr>
<td>7 8 9 10 11 12 0 1 2 3 4 5 6</td>
</tr>
</tbody>
</table>

5.4 How to Obtain Nearest Neighbor Balanced Design Using Rule II

Let \( S = [q_1, q_2, ..., q_{k-2}]^t \), where \( 1 \leq q_i \leq v-2 \). If each of 1, 2, ..., \( v-2 \) appears \( \lambda \) times in \( S^* \) then it is CNBD, where \( S^* = [q_1, q_2, ..., q_{k-1}, v-1-q_1, v-1-q_2, ..., v-1-q_{k-2}] \).

Example 5.4: Sets of shifts \([1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1] /1/11 + [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 0, 1, 2] \) provide minimal NBD for \( v = 12 \) and \( k = 11 \).
REFERENCES


Method of Cyclic Shifts for the Construction of Experimental Designs


