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Prediction Capability of Industrial Split-plot Response Surface Designs when One Observation is Missed

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Abstract

Response surface designs under restricted randomization or Split-plot response surface designs are often used in agriculture experiments and in industrial experiments due to existence of one or more factors that can't change their levels easily some factors need to estimate more precisely. The motive of this paper is to prevail prediction capability of a particular class of split-plot response surface designs, known as Central Composite Designs by Vining, Kowalski and Montgomery (VKM CCDs) when one observation of any category is missed. Both numerical and graphical methods, based on scaled prediction variance (SPV) are applied. Robustness of above class of designs against one missing observation is investigated relative to G-efficiency and Minimax loss designs are proposed. The prediction capability is computed by graphical methods such as 3D Variance Dispersion Graph (3D-VDG), Fraction of Design Space (FDS) plots and contour plots for extraordinary efficiency standards are used to look at the impact of lacking observations.

Keywords: Prediction Capability, G-efficiency, Scaled Prediction Variance, Fraction of Design space and Variance Dispersion Graph.

1. Introduction

Response Surface Methodology (RSM) is a type of experimental setup that may be used to create, improve, and optimize processes (Myers et al., 2009). VKM central composite designs (VKM CCD) are most commonly used second order split-plot response surface designs. Selection of a good experimental design and the corresponding model have been important tasks in any experimental setup. Particularly the selected design should be capable to predict the response. This prediction capability can be studied numerically as well as graphically. One of the criteria to compare experimental designs is G-optimality which states that a design is G-optimal if it minimizes the maximum scaled prediction variance (SPV) and graphical tools to study SPV includes Variance Dispersion Graph (VGD 2D & 3D), Fraction of Design space (FDS) and contour plots. The study of prediction capability becomes bit complicated when one or more observations are missed or excluded from analysis. The property of prediction capability is particularly desired in planned experiments of business and industry where future prediction is of intense importance. This paper possibly covers prediction capability of VKM CCD when an observation is missed. Prediction capability of this class of designs is invstigated using SPV and some well known graphical measures variance dispersion graphs, FDS plots and contour plots. The mathematical expressions of information matrix and other charactristics are simplified for VKM CCD. These expressions were presented by Wesley (2006). Finally VKM CCD robust to single missing observation are also recommended on the basis of G-efficiency. Some motivational examples are referred in next section to highlight the significance of this class of designs.

1.1 Motivational application of Spilt-plot Designs

Bingham et al. (2004). An experiment was once conducted in a cheese shop. The purpose of testing used to be to learn about some of the finer qualities of cheese making. The method of making cheese consists of two stages. In the first stage, the milk is turned into a batch of curds, and in the second stage, the curds are turned into cheese. Experts become aware of nine distinct elements that have an effect on quality. The qualities of cheese manufacturing below study. Two of these factors, say Z1 and Z2, are regarded as merchandise in the first phase, while the last factors, such as X1, X2, X3, X4, X5, X6, and X7 will produce on second step. It is endorsed to habits the test in two phases. First, massive portions of milk are processed in boilers at unique temperatures W1 and W2 coefficient settings. Processed milk from character boilers is then divide into batches of curd and manner these batches. Process into cheese at one-of-a-kind settings for the ultimate seven factors. It is an outstanding example of a split-plot designs W1 & W2 are whole plot factors and other seven factors are subplot factors.

Gilmour et al. (2000) elaborated the coffee freeze-drying experience. The cause of the test is to study effect of 5 elements on the crop reservation of risky compounds sharing costs. These 5 elements are: strain (Z1), solids content material (X1), Slab thickness (X2), temperature (X3) and solidification fee (X4) everything is take a look at on three distinctive levels. As we all know, the method will be varying a lot between scan runs, however there will be extra every day. The variety available sources enable up to 30 experiments run the scan and figure out to run it on 6 days, run 5 instances a day. However, it is no longer viable to run all 30 running absolutely randomized experiments with all tiers of elements within the time allowed due to the fact the stress thing (Z1) have to be changed. Manually, it takes a lengthy time to go from one stage to another. In different words, stress is an element for

HTC the experimenter wants Stressor ranges all through the day, then run randomly. Five exceptional degree settings for the ETC factor. A split-plot graph in which HTC thing stages are utilized to the wide variety of days (large experimental units) and the ETC element tiers are utilized to the collection (small experimental units) every day. The everyday entire pressure is whole plot factor while the closing four elements are subplot factors.

Jones and Goos (2007) stated an examination on polypropylene. The test used to be carried out by way of four Belgian organizations to learn about countless elements affecting the adhesive houses of polypropylene. The trouble studied is the gasoline plasma cure utilized to polypropylene surfaces so that glues and coatings can adhere well. Experimenters are generally involved in a low in cost plasma remedy that can impart correct adhesive residences to polypropylene. A basic instance of split-plot format is an irrigation experiment, the place irrigation degrees are utilized to a massive area, whilst elements such as range and fertilizer are assigned to smaller areas in a treatment distinct irrigation

2. Literature Review

Split-plot designs (SPD) had been at the start delivered for agricultural experiments and have when you consider that grow to be famous for industry-related experiments. These designs have obtained adequate interest in the literature in current years. Letsinger et al. (1996) furnished RSMs for estimating SP designs and counseled generalized least squares for estimating SPRS models. Bingham and Sitter (1999) and Bingham et al. (2004) supplied a structural layout of 2k-p SP the use of aberration criteria. Trinca and Gilmour (2001) proposed a multi-layered sketch primarily based on sequential methods. Vining et al. (2005) furnished the SP central composite layout (VKM CCD) and the SP Box-Behnken format (VKM BBD) with the extended central composite sketch and the thoroughly random structured Box-Behnken design. These plans are generalized varieties of split-plot plans. Kulahci and Bisgaard (2005) give an explanation for how to create SP designs from Plackett-Burman designs. Yang et al. (2007) gave a number of outcomes for establishing fractional factorial SP designs with low minimal aberrations.

Goos and Vandebroek (2001, 2003 and 2004) and Jones and Goos (2007) proposed to assemble SP D-optimal plans the use of the swap algorithm. See Jones and Nachtsheim (2009) for an overview of current traits in SP experimental design. For estimation of second-order SP response floor mannequin parameters, we describe designs for which the OLS and GLS estimators of some mannequin parameters are equivalent, and regard these designs as equal estimating SP designs and generic necessities for this property. They additionally provide catalogs for VKM CDD and VKM BBD primarily based on designs by using Center Composites and Box-Behnken designs. Parker et al. (2006, 2007a, 2007b) proposed two strategies to assemble SP designs the usage of balanced equivalence estimates and prolonged these strategies to assemble SP designs with unbalanced equivalence estimates. Liang (2005) adopted two graphical tools, 3D VDG and FDS, for SPRS design. SP designs have obtained big attention, however the robustness of these designs to lacking observations has now not been noted, without in the article with the aid of Chukwu et al. (2013). He researches the loss of special lacking observations related with CCD defects. Gandhi and Kumaran (2014) Optimization of biodiesel information research the usage of RSM. He makes use of these numbers to find out about and evaluate one-of-a-kind CCD variants.

Yakubu et al. (2014) investigated the impact of missing observations on the predictive power and accuracy of CCD estimation in a definitely random shape (CRD). Srisuradetchai (2015) investigated the robustness of CRD response floor designs to lacking values. He additionally added effective metrics and developed R packages. Iwundu (2017) investigates the loss of one or two lacking values in response floor designs associated to D, A, and efficiency. Alrweill et al. (2019) assembles strong response floor designs that are extra sturdy to single observations than present authentic designs. The s trendy has its limits. A trouble is that if these runs are inadequate to estimate the parameter of interest, they can't be utilized to the last runs. Another quandary is that every criterion focuses on a region of interest, such as loss, D yield, estimation power, etc. Therefore, greater bendy sturdy statement standards may additionally be wished to stay away from these limitations. Oladugba and Ossai (2020) derived from a non-iterative least-squares method to estimate a couple of lacking records in rectangular lattice designs (single and triple rectangular lattice designs) except repeating the base plan the usage of within-block information. The non-iterative least squares approach minimizes the intra-block sum of the squared blunders with recognize to the lacking facts and solves the end result to reap an estimate of the lacking data. Yankam and Oladugba (2023) constructed an Orthogonal Uniform Composite Loss-Max-min (OUCM) design. They compared these designs with the core composite design, small composite design, orthogonal array composite design, and orthogonal array minimum loss maximum composite design and found that the OUCM design performed better in loss and also had better High D, E and T efficiencies.

3. Research Design

Methodology consists of Second Order Response Surface (SORS) design approaches and their characteristics, losses because of missing observations related to various efficiency criteria, and sub-classes of the design to be investigated. Consider a split-plot (SP) design with w extensive variety of whole plot elements and k extensive variety of subplot elements. Let $\boldsymbol{N = \sum_{i=1}^{m} n_i}$ be the total runs in a design, in which m is to be available units for whole plots elements and n_i the dimensions of i_{th} whole plot. For balanced design n_i=n then the overall shape of the variety reading the statistics of this test.

$$\boldsymbol{y = X\beta + \delta + \epsilon} \qquad \textbf{(i)}$$

Where vector **y** is of response of order **N×1, X** is the **N×p** matrix of coefficients (version matrix), **β** is the **p×1** vector of parameters, **δ** is the random vector of entire plot inaccuracies of order $\mathbf{N\times 1}$ and $\boldsymbol{\epsilon}$ is the random vector of sub-plot errors. The standard shape of the version vector is given as.

$$\boldsymbol{f}(\boldsymbol{z},\mathbf{x})' = [\mathbf{1}|\mathbf{z_1},\ldots,\mathbf{z}_w|\mathbf{x_1},\ldots,\mathbf{x}_k|\mathbf{z_1z_2},\ldots,\mathbf{z}_{w-1}\mathbf{z}_w|\mathbf{z_1x_1},\ldots,\mathbf{z}_w\mathbf{x}_k|\mathbf{x_1x_2},\ldots,\mathbf{x}_{k-1}\mathbf{x}_k|\boldsymbol{z}_1^2,\ldots,\boldsymbol{z}_w^2|\mathbf{x}_1^2,\ldots,\mathbf{x}_k^2] \quad \textbf{(ii)}$$

Where **z** and **x** represent the coefficients of the whole plot and subplot. The model (1) was previously assumed $\boldsymbol{\delta}+\boldsymbol{\epsilon}$ to contain zero mean and variance and covariance matrices.

$$\boldsymbol{\Sigma} = \boldsymbol{\sigma}_{\epsilon}^2\boldsymbol{I} + \boldsymbol{\sigma}_{\delta}^2\mathbf{J_b} \qquad \textbf{(iii)}$$

Where $\boldsymbol{\sigma}_{\epsilon}^2$ is the variance due to subplot error and $\boldsymbol{\sigma}_{\delta}^2$ is the variance whole-plot error. For balanced design, the $N \times N$ matrix is given as $\mathbf{J_b} = \mathbf{I}_m \otimes \mathbf{J}_n$or equivalently. The goal of the researcher may additionally be to make attractive predictions in precise with in the design space. To do this, Box and Hunter (1957) described a variance function, which is also called the scaled prediction variance (SPV). SPV offers the accuracy of the estimated response at any point in the design space.

$$\boldsymbol{SPV} = \boldsymbol{N}\mathbf{x}_0'(\boldsymbol{X}'\mathbf{R}^{-1}\boldsymbol{X})^{-1}\mathbf{x_0} \qquad \textbf{(iv)}$$

The objective of this study is to find the robustness of our purposed class of design in the terms of losses in efficiency for Robust VKM CCD are computed by the following rules which used when we missed single design points in every design point. The relative loss in G-efficiency (l_1) due to missing single observation for individually category of design points, diverse values of variance ratio (1, 5, & 10) and α, β are calculated by

$$\textbf{Relative Loss in G-efficiency} \quad \boldsymbol{l_3} = \frac{\boldsymbol{Max_{x_0\epsilon R}SPV}}{(\boldsymbol{Max_{x_0\epsilon R}SPV})_r} - \mathbf{1} \qquad \textbf{(v)}$$

4. Findings and Discussion

Vining et al. (2005) proposed a class of second-order response surface designs under the split graph structure, called central composite VKM designs. The name "VKM" for this type of design is due to the authors Vining, Kowalski and Montgomery. An important property of these designs is that ordinary least squares parameter estimation is considered the same as generalized least squares for such models. Due to this property, these plans are also called equivalent estimate plans. These designs are very efficient in saving experimental resources and are easily used for whole-plot and pure-error subplot error variance estimation. The notation D (w; k) denotes a design with w whole-plot factors and k sub-plot factors. Split-plot response surface designs with 1<w<3 and 1<k<4 is most commonly used for applications. This paper also considers such designs with the same number of factors to study their robustness to missing values. VKM CCD is a split-plot version of the CCD. The total number of factors for these central composite planes is F = w + k. A is the distance between the center of the subplot factor and the axis/star point, and β is the distance between the center of the WP factor and the axis/star point. It is important to note that the values of α and β are considered the same when calculating the loss and the survey. This study aims to find the loss of a missing value in the VKM CCD. WP factor ≤ 3 and subplot factor ≤ 4. Consider different but identical values of α and β. Another factor is the variance ratio (VR), which includes three values of 1, 5 and 10. The effect of missing values is calculated for the four types of design points and for different values of α, β and the ratio variance. In this research offered the overall expressions of information matrices, its inverse, determinant and hint changed for VKM CCD to similarly speak the robustness of this magnificence of designs and additionally assemble for two factor VKM CCD (1, 1). The Mathematical expressions of information matrix at the side of a few important features for three-factor VKM CCD (1, 2) and Four-factor (1, 3) are given in whole description and other designs are summary and discussion.

4.1. Robust Three-factor VKM CCD (1, 2)

The three-factor VKM CCD (1, 2) includes one whole-plot and two sub-plot factors. This design has total 24 design points, including eight factorial points, eight WP axial points, four sub plot axial and four center points. For orthogonal and rotatable design points values are (0.9555) and (1.4142) respectively. The losses due to missing each design point are calculated relative to A, D and G-efficiency for diverse range of α, β values and fixed variance ratio in which the three factor VKM CCD (1, 2) design is orthogonal and rotatable. So, this design is called robust three-factor VKM CCD (1, 2) against single missing observation. The results are mentions in tables below and the last column of these tables shows maximum loss for missing single value and figures of results are also constructed for maximum losses.

$$\mathrm{R}_i = \begin{pmatrix} 1 & \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} & 1 & \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} & 1 & \frac{\eta}{1+\eta} \\ \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} & \frac{\eta}{1+\eta} & 1 \end{pmatrix}$$

$$
= \begin{bmatrix}
1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -1 & 1 & -1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & -1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & -1 & 1 & 1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & -1 & -1 & -1 & -1 & 1 & 1 & 1 & 1 \\
1 & 1 & 1 & -1 & 1 & -1 & -1 & 1 & 1 & 1 \\
1 & 1 & -1 & 1 & -1 & 1 & -1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\
1 & -\beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & -\beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & -\beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & -\beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & \beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & \beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & \beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & \beta & 0 & 0 & 0 & 0 & 0 & \beta^2 & 0 & 0 \\
1 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & \alpha^2 & 0 \\
1 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \alpha^2 & 0 \\
1 & 0 & 0 & -\alpha & 0 & 0 & 0 & 0 & 0 & \alpha^2 \\
1 & 0 & 0 & \alpha & 0 & 0 & 0 & 0 & 0 & \alpha^2 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Information Matrix for Robust VKM CCD (1, 2) is given below:

$(X'R^{-1}X)$

$$
= \begin{bmatrix}
\Pi & 0 & \frac{8(1+\beta^2)(1+\eta)}{1+4\eta} & \frac{2(4+\alpha^2)(1+\eta)}{1+4\eta} \\
0 & \text{Diag (di)} & 0 & 0 \\
\frac{8(1+\beta^2)(1+\eta)}{1+4\eta} & 0 & \frac{8(1+\beta^4)(1+\eta)}{1+4\eta} & \frac{8(1+\eta)}{1+4\eta} \\
\frac{2(4+\alpha^2)(1+\eta)}{1+4\eta} & 0 & \frac{8(1+\eta)}{1+4\eta} & \frac{2(1+\eta)(4+\alpha^4(1+2\eta))}{1+4\eta}
\end{bmatrix}
$$

Determinant of Information Matrix for Robust VKM CCD (1, 2)

$$
|X'R^{-1}X| = \frac{\mathbf{2097152(4+\alpha^2)^2(1+\beta^2)(16\alpha^4\beta^4-8\alpha^6\beta^2(-1+\beta^2)+\alpha^8(3-4\beta^2+3\beta^4))(1+\eta)^{10}}}{\mathbf{(1+4\eta)^4}}
$$

4.2 Robust Four-factor VKM CCD (1, 3)

The four-factor VKM CCD (1, 3) includes one whole-plot and three sub-plot factors. This design has total 64 design points, including twenty-four factorial points, twenty-four WP axial points, eight sub plot axial and eight center points. For orthogonal and rotatable design points values are (0.94777) and (1.4142) respectively. The losses due to missing each design point are calculated relative to A, D and G-efficiency for diverse range of α, β values and fixed variance ratio in which the four-factor VKM CCD (1, 3) design is orthogonal and rotatable. So, this design is called robust four-factor VKM CCD (1, 3) against single missing observation.

$$(X'R^{-1}X) = \begin{bmatrix} \Pi & 0 & \frac{16(1+\beta^2)(1+\eta)}{1+8\eta} & \frac{8(2+\alpha^2)(1+\eta)}{1+8\eta} \\ 0 & \text{Diag (di)} & 0 & 0 \\ \frac{16(1+\beta^2)(1+\eta)}{1+8\eta} & 0 & \frac{16(1+\beta^4)(1+\eta)}{1+8\eta} & \frac{16(1+\eta)}{1+8\eta} \\ \frac{8(4+\alpha^2)(1+\eta)}{1+8\eta} & 0 & \frac{16(1+\eta)}{1+8\eta} & \frac{8(2+\alpha^4)(1+\eta)}{1+8\eta} \end{bmatrix}$$

$|X'R^{-1}X|=$

$$\frac{2097152a^4(1+b^2)(1+c)^{10}(256b^4(1+4c)+5a^6(3-4b^2+3b^4)(1+4c)+ a^4(48(1+4c)-24b^2(1+4c)+b^4(9+32c))-4a^2(-32b^2(1+4c)+b^4(11+48c)}{(1+4c)^5}$$

Table 1: Comparison between Three-factor & Four -factor VKM CDD

Robust VKM CCD (1, 2)	**Robust VKM CCD (1, 3)**
Number of total runs (N=24)	Number of total runs (N=64)
$\Pi = N\Delta_1 = \frac{24(1+\eta)}{(1+2\eta)}$	$\Pi = N\Delta_1 = \frac{64(1+\eta)}{(1+2\eta)}$
Factorial points (*f*= 8)	Factorial points (*f*= *24*)
WP axial points (f_w = 8)	WP axial points (f_w = 24)
Whole-plot factors (w_p = 1)	Whole-plot factors (w_p = 1)
Sub-plot factor (s_p = 2)	Sub-plot factor (s_p = 3)
Sub plot axial (w_i = 4)	Sub plot axial (w_i = 8)
Center points r_w = 4	Center points (r_w = 8)
Orthogonality = 0.9555	Orthogonality = 0.94777
Rotatability = 1.4142	Rotatability = 1.4142
Partitioning of Information Matrix for Robust VKM CCD (1, 2 & 1, 3)	
A = $\begin{bmatrix} \boldsymbol{A_{11}} & \boldsymbol{A_{12}} \\ \boldsymbol{A_{21}} & \boldsymbol{A_{22}} \end{bmatrix}$,	**B** = $\begin{bmatrix} \boldsymbol{B_{11}} & \boldsymbol{B_{12}} \\ \boldsymbol{B_{21}} & \boldsymbol{B_{22}} \end{bmatrix}$,
$\mathbf{A_{11}} = \begin{bmatrix} \boldsymbol{\Pi} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Diag(d_i)} \end{bmatrix}$	$\mathbf{B_{11}} = \begin{bmatrix} \boldsymbol{\Pi} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{Diag(d_i)} \end{bmatrix}$,
$\mathbf{A_{12}} = \begin{bmatrix} \frac{8(1+\beta^2)(1+\eta)}{1+4\eta} & \frac{2(4+\alpha^2)(1+\eta)}{1+4\eta} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\mathbf{B_{12}} = \begin{bmatrix} \frac{16(1+\beta^2)(1+\eta)}{1+8\eta} & \frac{8(2+\alpha^2)(1+\eta)}{1+8\eta} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$,
$\mathbf{A_{21}} = \begin{bmatrix} \frac{8(1+\beta^2)(1+\eta)}{1+4\eta} & \mathbf{0} \\ \frac{2(4+\alpha^2)(1+\eta)}{1+4\eta} & \mathbf{0} \end{bmatrix}$	$\mathbf{B_{21}} = \begin{bmatrix} \frac{16(1+\beta^2)(1+\eta)}{1+8\eta} & \mathbf{0} \\ \frac{8(4+\alpha^2)(1+\eta)}{1+8\eta} & \mathbf{0} \end{bmatrix}$
$\mathbf{A_{22}} = \begin{bmatrix} \frac{8(1+\beta^4)(1+\eta)}{1+4\eta} & \frac{8(1+\eta)}{1+4\eta} \\ \frac{8(1+\eta)}{1+4\eta} & \frac{2(1+\eta)(4+\alpha^4(1+2}{1+4\eta} \end{bmatrix}$	$\mathbf{B_{22}} = \begin{bmatrix} \frac{16(1+\beta^4)(1+\eta)}{1+8\eta} & \frac{16(1+\eta)}{1+8\eta} \\ \frac{16(1+\eta)}{1+8\eta} & \frac{8(2+\alpha^4)(1+2\eta)}{1+8\eta} \end{bmatrix}$

Table 1 shows the complete comparison between the three-factor robust VKM CCD (1, 2) and four-factor robust VKM CCD (1,3) by mathematically computations of both designs also compare the Partitioning of the information matrix for VKM CCD (1, 2 & 1, 3).

4.3 Robust Comparison in G-efficiency between Three-factor (1,2) & Four-factors (1,3, 2, 2 & 3,1) VKM CCD

In this section compare the three-factor and four-factor designs based on relative loss in G-efficiency (l_3) due to missing single observation for individually category of design points, diverse values of variance ratio (1, 5, & 10) and α, β are given in table (2, 3 & 4) respectively and figure 1 shows the relative loss in G-efficiency has decreasing trend at different values of variance ratio and α, β. Similarly, in figure 2 shows also decreasing trend in overall maximum losses for three-factor VKM CCD. Result in table 2 present the robustness for robust VKM CCD (1, 2), when variance ratio (V.R) 1 and the design with α=β=2.5 shows minimax loss due to relative G-efficiency, i.e. (0.06657). Similarly, when V.R is 5 and 10 minimax losses due to relative G-efficiency, i.e. (0.01236 & 0.01236) are at α=β=2.5 respectively. Overall, minimax loss due to relative G-efficiency, i.e. (0.00634) at variance ratio 10 and α=β=2.5 The robust three-factor design VKM CCD (1, 2) is called relative to G-efficient.

Table 2: Loss in G-efficiency for Robust VKM CCD (1, 2)

Variance Ratio	Alpha/ βeta	when missing single value in				Max Loss
		FP	WP	SP	CP	
1	0.5	0.10766	0.00197	0.00485	0.00004	0.10766
	0.7	0.14023	0.00301	0.01048	0.00016	0.14023
	0.9	0.17442	0.00388	0.01964	0.00045	0.17442
	1.1	0.20496	0.00468	0.03454	0.00094	0.20496
	1.3	0.22825	0.00561	0.05892	0.00133	0.22825
	1.5	0.23087	0.00017	0.08236	0.06143	0.23087
	1.7	0.03984	0.00000	0.0001	0.06657	0.06657*
	1.9	0.11563	0.00049	0.08414	0.00074	0.11563
	2.1	0.08778	0.00022	0.0853	0.00172	0.08778
	2.3	0.06753	0.00009	0.08515	0.00194	0.08515
	2.5	0.05372	0.00004	0.08282	0.00174	0.08282
5	0.5	0.03599	0.00065	0.00162	0.00001	0.03599
	0.7	0.04822	0.00101	0.00357	0.00005	0.04822
	0.9	0.06164	0.00132	0.00683	0.00015	0.06164
	1.1	0.07466	0.00162	0.01237	0.00033	0.07466
	1.3	0.00658	0.01236	0.00073	0.0046	0.01236*
	1.5	0.00072	0.00004	0.00154	0.01468	0.01468
	1.7	0.00002	0.00000	0.00003	0.01585	0.01585
	1.9	0.00046	0.00001	0.00053	0.01535	0.01535
	2.1	0.00347	0.01414	0.00021	0.00003	0.01414
	2.3	0.02384	0.00003	0.02816	0.00063	0.02816
	2.5	0.01879	0.00001	0.02705	0.00056	0.02705
10	0.5	0.01961	0.00036	0.00088	0.00001	0.01961
	0.7	0.02641	0.00055	0.00195	0.00003	0.02641
	0.9	0.03395	0.00072	0.00375	0.00008	0.03395
	1.1	0.03304	0.00604	0.0003	0.00012	0.03304
	1.3	0.00341	0.00634	0.00038	0.00008	0.00634**
	1.5	0.00037	0.00002	0.00079	0.00752	0.00752
	1.7	0.00001	0.00000	0.00001	0.00812	0.00812
	1.9	0.00024	0.00001	0.00027	0.00786	0.00786
	2.1	0.0018	0.00725	0.00011	0.00002	0.00725
	2.3	0.0014	0.00739	0.00005	0.00002	0.00739
	2.5	0.0103	0.00001	0.01467	0.00031	0.01467

*Minimax loss for fixed variance ratio **Overall Minimax loss

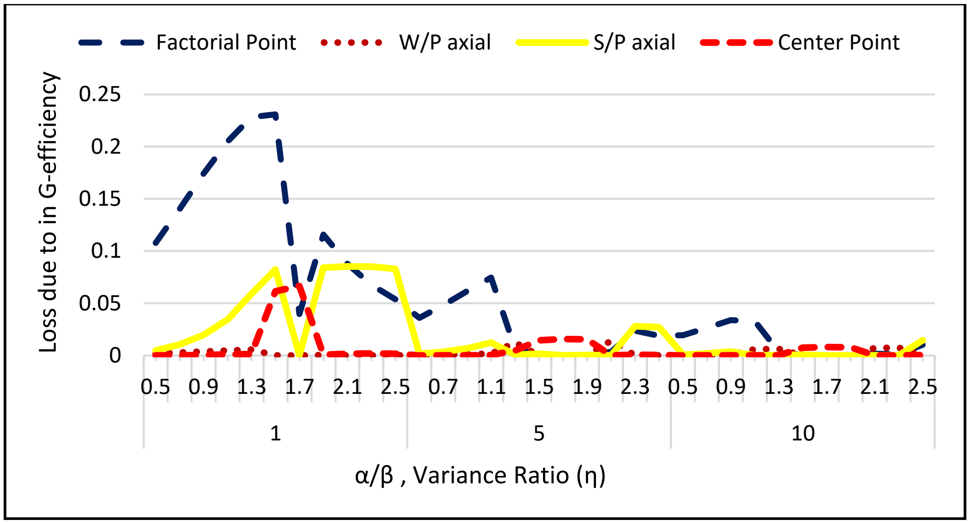

Figure 1. Loss in G-efficiency for Robust VKM CCD (1, 2)

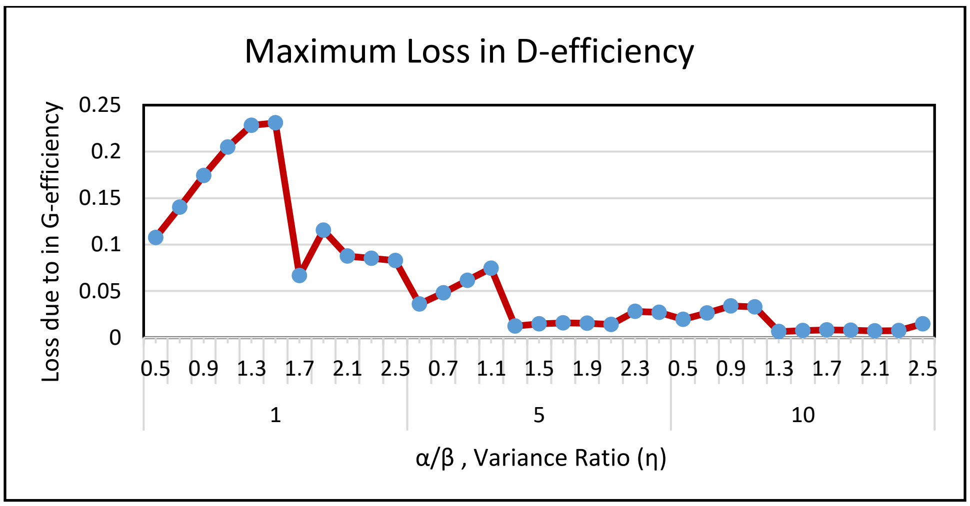

Figure 2. Maximum Loss in G-efficiency for Robust VKM CCD (1, 2)

Robustness of robust VKM CCD (1, 3) shows in table 3, when variance ratio (V.R) 1 and the design with α=β=1.9 shows minimax loss due to relative G-efficiency, i.e. (0.03022). Similarly, when V.R is 5 and 10 minimax losses due to relative G-efficiency, i.e. (0.00343

& 0.00174) are at α=β=1.9 respectively. Overall, minimax loss due to relative G-efficiency, i.e. (0.00174) at variance ratio 10 and α=β=1.9 The robust four-factor design VKM CCD (1, 3) is called relative to G-efficient.

Table 3: Loss in G-efficiency for VKM CCD (1, 3)

Variance Ratio	Alpha/ βeta	when missing single value in				Max Loss
		FP	WP	SP	CP	
1	0.5	0.09371	0.00047	0.00061	0.00001	0.09371
	0.7	0.11628	0.00072	0.0011	0.00004	0.11628
	0.9	0.14197	0.0009	0.00166	0.00011	0.14197
	1.1	0.16867	0.00105	0.00228	0.00027	0.16867
	1.3	0.19559	0.00122	0.00308	0.00055	0.19559
	1.5	0.19516	0.00005	0.03639	0.00062	0.19516
	1.7	0.04941	0.00005	0.03969	0.00026	0.04941
	1.9	0.00015	0.00001	0.03022	0.01563	0.03022*
	2.1	0.00743	0.00002	0.04568	0.00115	0.04568
	2.3	0.00605	0.00001	0.04818	0.00007	0.04818
	2.5	0.00488	0.00000	0.0503	0.0001	0.0503
5	0.5	0.02836	0.00014	0.00019	0.00000	0.02836
	0.7	0.0272	0.00000	0.00489	0.00018	0.0272
	0.9	0.00108	0.00000	0.00575	0.00019	0.00575
	1.1	0.00132	0.00001	0.00665	0.0002	0.00665
	1.3	0.00158	0.00001	0.00755	0.00019	0.00755
	1.5	0.00186	0.00001	0.00841	0.00014	0.00841
	1.7	0.00207	0.00001	0.00922	0.00006	0.00922
	1.9	0.00003	0.00000	0.00001	0.00343	0.00343*
	2.1	0.00003	0.00000	0.00000	0.00344	0.00344
	2.3	0.00144	0.00000	0.01127	0.00002	0.01127
	2.5	0.00117	0.00000	0.01179	0.00002	0.01179
10	0.5	0.01515	0.00008	0.0001	0.00000	0.01515
	0.7	0.00041	0.00000	0.00248	0.00009	0.00248
	0.9	0.00055	0.00000	0.00292	0.0001	0.00292
	1.1	0.00067	0.00000	0.00339	0.0001	0.00339
	1.3	0.0008	0.00000	0.00385	0.0001	0.00385
	1.5	0.00095	0.00001	0.00429	0.00007	0.00429
	1.7	0.00106	0.00001	0.00471	0.00003	0.00471
	1.9	0.00002	0.00000	0.00000	0.00174	0.00174**
	2.1	0.00002	0.00000	0.00000	0.00174	0.00174
	2.3	0.00074	0.00000	0.00576	0.00001	0.00576
	2.5	0.0006	0.00000	0.00602	0.00001	0.00602

*Minimax loss for VKM CCD fixed variance ratio **Overall Minimax loss

The four-factor VKM CCD (2, 2) includes two whole-plot and two sub-plot factors. This design has total 40 design points, including sixteen factorial points, sixteen WP axial points, four sub plot axial and four center points. For orthogonal and rotatable design points values are (0.9568) and (1.4142) respectively. The losses due to missing each design point are calculated relative G-efficiency for diverse range of α, β values and fixed variance ratio in which the four-factor VKM CCD (2, 2) design is orthogonal and rotatable. So, this design is called robust four-factor VKM CCD (2, 2) against single missing observation. The four-factor VKM CCD (3, 1) includes three whole-plot and one sub-plot factors. This design has total 32 design points, including 16 factorial points, 12 WP axial points, two sub plot axial and two center points. For orthogonal and rotatable design points values are (0.9568) and (1.4142) respectively. So, this design is called robust four-factor VKM CCD (3, 1) against single missing observation.

For this robust VKM CCD (2, 2), when variance ratio (V.R) 1 and the design with α=β=1.9 shows minimax loss due to relative G-efficiency, i.e. (0.00009). Similarly, when V.R is 5 and 10 minimax losses due to relative G-efficiency, i.e. (0.00029 & 0.00014) are at α=β=0.5 respectively. Overall, minimax loss due to relative G-efficiency, i.e. (0.00009) at variance ratio 10 and α=β=1.9 The robust four-factor design VKM CCD (2, 2) is called relative to G-efficient. For this robust VKM CCD (3, 1), when variance ratio (V.R) 1 and the design with α=β=2.3 shows minimax loss due to relative G-efficiency, i.e. (0.11453). Similarly, when V.R is 5 and 10 minimax losses due to relative G-efficiency, i.e. (0.03635 & 0.01959) are at α=β=2.3 respectively. Overall, minimax loss due to relative G-efficiency, i.e. (0.01959) at variance ratio 10 and α=β=2.3. The robust four-factor design VKM CCD (3, 1) is called relative to G-efficient.

Table 4: Summary for loss in G-efficiency for Robust VKM CCD (2, 2 & 3, 1)

VR	α/β	When missing single value in				Max. Loss
		FP	WP	SP	CP	
G-efficiency for Four-factor VKM CCD (2, 2).						
1	1.9	0.00009	0.00000	0.00008	0.00000	0.00009**
5	0.5	0.00000	0.00029	0.00009	0.00029	0.00029*
10	0.5	0.00000	0.00014	0.00006	0.00014	0.00014*
G-efficiency for Four-factor VKM CCD (3, 1)						
1	2.3	0.03283	0.00131	0.11453	0.00879	0.11453*
5	2.3	0.01153	0.00042	0.03635	0.00279	0.03635*
10	2.3	0.00634	0.00023	0.01959	0.0015	0.01959**

*Minimax loss for fixed variance ratio **Overall Minimax loss

4.4 Loss in G-efficiency for Robust five-factor VKM CCD (1, 4) and Robust Six-Factor (3, 3)

The five-factor VKM CCD (1, 4) includes one whole-plot and four sub-plot factors. This design has total 48 design points, including sixteen factorial points, sixteen WP axial points, eight sub plot axial and eight center points. For orthogonal and rotatable design points values are (1.04834) and (1.68179) respectively. The losses due to missing each design point are calculated relative to G-efficiency for diverse range of α, β values and fixed variance ratio in which the five-factor VKM CCD (1, 4) design is orthogonal and rotatable. So, this design is called robust five-factor VKM CCD (1, 4) against single missing observation. The results are mentions in table 4 which is consisting of over-all summary for loss in G-efficiency for Robust augmented (1, 4). For this robust VKM CCD (1, 4), when variance ratio (V.R) 1 and the design with α=β=0.5 shows minimax loss due to relative G-efficiency, i.e. (0.00111). Similarly, when V.R is 5 and 10 minimax losses due to relative G-efficiency, i.e. (0.00038 & 0.00021) are at α=β=0.5. Overall, minimax loss due to relative G-efficiency, i.e. (0.00111) at variance ratio 1 and α=β=0.5. The robust five-factor design VKM CCD (1, 4) is called relative to G-efficient.

The six-factor VKM CCD (3, 3) includes three whole-plot and three sub-plot factors. This design has total 72 design points, including thirty-two factorial points, thirty-two WP axial points, four sub plot axial and four center points. For orthogonal and rotatable design points values are (1.09757) and (2.37841) respectively. The losses due to missing each design point are calculated relative to G-efficiency for diverse range of α, β values and fixed variance ratio in which the six-factor VKM CCD (3, 3) design is orthogonal and rotatable. So, this design is called robust six-factor VKM CCD (3, 3) against single missing observation. The results are mentions in tables 5. For this robust VKM CCD (3, 3), when variance ratio (V.R) 1 and the design with α=β=1.1 shows minimax loss due to relative G-efficiency, i.e. (0.00157). Similarly, when V.R is 5 and 10 minimax losses due to relative G-efficiency, i.e. (0.01677& 0.0086) are at α=β=0.5 respectively. Overall, minimax loss due to relative G-efficiency, i.e. (0.0086) at variance ratio 10 and α=β=0.5. The robust six-factor design VKM CCD (3, 3) is called relative to G-efficient.

Table 5: G-efficiency for Robust Five-factor VKM CCD (1, 4).

Variance Ratio	Alpha/βeta	when missing single value in				Max. Loss
		FP	WP	SP	CP	
1	0.5	0.00009	0.00043	0.00111	0.00000	0.00111*
	0.7	0.00027	0.00067	0.00233	0.00001	0.00233
	0.9	0.0006	0.00085	0.00415	0.00002	0.00415
	1.1	0.00105	0.00097	0.00669	0.00004	0.00669
	1.3	0.00159	0.00103	0.01014	0.00007	0.01014
	1.5	0.00215	0.00105	0.01479	0.00011	0.01479
	1.7	0.00268	0.00105	0.02094	0.00013	0.02094
	1.9	0.00312	0.00103	0.02871	0.0001	0.02871
	2.1	0.00347	0.00101	0.03751	0.00003	0.03751
	2.3	0.00384	0.00096	0.04588	0.0095	0.04588
	2.5	0.01233	0.00001	0.17665	0.00041	0.17665
5	0.5	0.00003	0.00015	0.00038	0.00000	0.00038*
	0.7	0.0001	0.00024	0.00083	0.00000	0.00083
	0.9	0.00022	0.00031	0.00152	0.00001	0.00152
	1.1	0.0004	0.00036	0.00252	0.00002	0.00252
	1.3	0.00016	0.0027	0.0015	0.00001	0.0027
	1.5	0.00014	0.00282	0.00003	0.00001	0.00282
	1.7	0.00012	0.00292	0.00002	0.00000	0.00292
	1.9	0.00003	0.00000	0.00052	0.00319	0.00319
	2.1	0.00001	0.00000	0.00009	0.00343	0.00343
	2.3	0.00000	0.00000	0.00002	0.00347	0.00347
	2.5	0.00004	0.00000	0.00026	0.00331	0.00331
10	0.5	0.00002	0.00008	0.00021	0.00000	0.00021**
	0.7	0.00005	0.00013	0.00046	0.00000	0.00046
	0.9	0.00012	0.00017	0.00085	0.00000	0.00085
	1.1	0.00022	0.0002	0.00141	0.00001	0.00141
	1.3	0.00008	0.00137	0.00001	0.00000	0.00137
	1.5	0.00007	0.00143	0.00001	0.00000	0.00143
	1.7	0.00006	0.00148	0.00001	0.00000	0.00148
	1.9	0.00002	0.00000	0.00026	0.00161	0.00161
	2.1	0.00000	0.00000	0.00004	0.00174	0.00174
	2.3	0.00000	0.00000	0.00001	0.00176	0.00176
	2.5	0.00002	0.00000	0.00013	0.00168	0.00168

*Minimax loss for fixed variance ratio **Overall Minimax loss

Table 6. G-efficiency for Robust Six-factor VKM CCD (3, 3)

Variance Ratio	Alpha/ βeta	when missing single value in				Max. Loss
		FP	WP	SP	CP	
1	0.5	0.00858	0.00016	0.00016	0.00013	0.00858
	0.7	0.00626	0.00029	0.00029	0.00026	0.00626
	0.9	0.00314	0.00043	0.00043	0.00041	0.00314
	1.1	0.00157	0.00057	0.00057	0.00059	0.00157*
	1.3	0.00152	0.00069	0.00069	0.01779	0.01779
	1.5	0.00146	0.00078	0.00078	0.05405	0.05405
	1.7	0.00141	0.00086	0.00086	0.09184	0.09184
	1.9	0.00082	0.00000	0.00000	0.11118	0.11118
	2.1	0.00088	0.00000	0.00000	0.11933	0.11933
	2.3	0.00093	0.00000	0.00000	0.12769	0.12769
	2.5	0.00096	0.00000	0.00000	0.13613	0.13613
5	0.5	0.00003	0.00000	0.00000	0.01677	0.01677*
	0.7	0.00005	0.00000	0.00000	0.01763	0.01763
	0.9	0.00008	0.00000	0.00000	0.01874	0.01874
	1.1	0.00011	0.00000	0.00000	0.02011	0.02011
	1.3	0.00014	0.00000	0.00000	0.0217	0.0217
	1.5	0.00016	0.00000	0.00000	0.02348	0.02348
	1.7	0.00019	0.00000	0.00000	0.02544	0.02544
	1.9	0.00021	0.00000	0.00000	0.02753	0.02753
	2.1	0.00023	0.00000	0.00000	0.02972	0.02972
	2.3	0.00024	0.00000	0.00000	0.03198	0.03198
	2.5	0.00025	0.00000	0.00000	0.03428	0.03428
10	0.5	0.00001	0.00000	0.00000	0.0086	0.0086**
	0.7	0.00003	0.00000	0.00000	0.00904	0.00904
	0.9	0.00004	0.00000	0.00000	0.00962	0.00962
	1.1	0.00005	0.00000	0.00000	0.01033	0.01033
	1.3	0.00007	0.00000	0.00000	0.01115	0.01115
	1.5	0.00009	0.00000	0.00000	0.01208	0.01208
	1.7	0.0001	0.00000	0.00000	0.0131	0.0131
	1.9	0.00011	0.00000	0.00000	0.01419	0.01419
	2.1	0.00012	0.00000	0.00000	0.01533	0.01533
	2.3	0.00013	0.00000	0.00000	0.01651	0.01651
	2.5	0.00013	0.00000	0.00000	0.01771	0.01771

*Minimax loss for fixed variance ratio **Overall Minimax loss

4.5 Graphical Measures with Prediction Capabilities

In three factor VKM CCD (1, 2) the variance dispersion graph (2D-VGD & 3D-VGD) constructed for different values of α/β = [0.5,0.7,0.9,1.1,1.3,1.5,1.7,1.9,2.1,2.3,2.5] and fixed variance ratio (1,5,10), of designs points against the maximum scaled prediction variance (SPV). The two-dimensional (2D-VDG) is given in figure 5 whereas three-dimensional Variance Dispersions Graph (3D-VDG) for variance ratio (1, 5 & 10) in figures (3, 4, 5 & 6) respectively. In this section the three-factor VKM CCD (1, 2) shown by graphically representation on the basis of the above 2D- and 3D-VDGs figures, may be explained. When the values of SPV at different values of variance ratio and α, β, has a curvature in 2D graphs. When Variance ratio (VR) = 1, the design with α=β=0.5 shows the maximum SPV, i.e., 0.8668 whereas at α=β=1.5, SPV has a minimum value 0.5787. Similarly, when VR is 5 & 10, the minimum values (0.6837, 0.6933) of SPV are at α=β=1.3 and 1.1 respectively, and have maximum values (0.8880, 0.9305) are at α=β=0.5& 1.7 respectively. Overall, the maximum SPV is minimum at VR=1, α=β=1.5 and maximum at VR=10, α=β=1.7. In 3D- VDG at different values of variance ratio are given and colure bar shows its trend on variant values of alpha and beta.

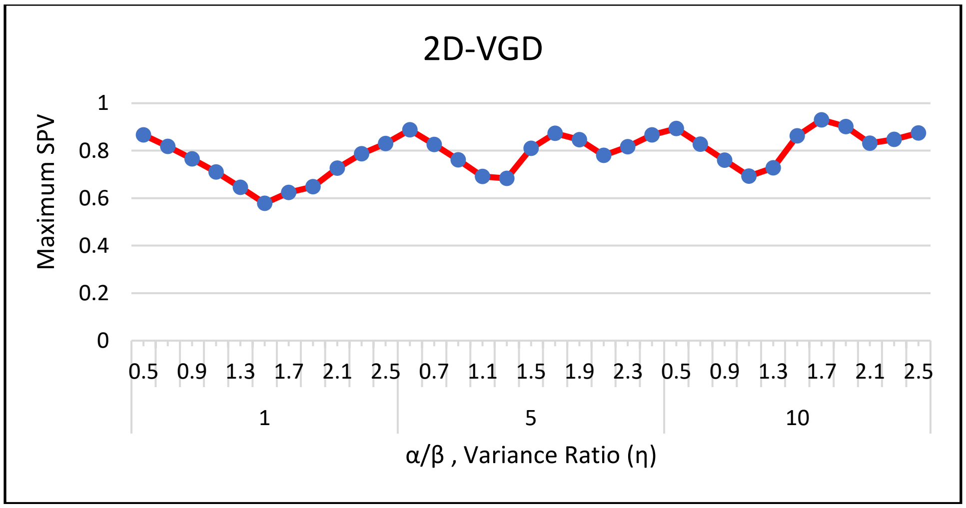

Figure 3: 2D-VDG for VKM CCD (1, 2)

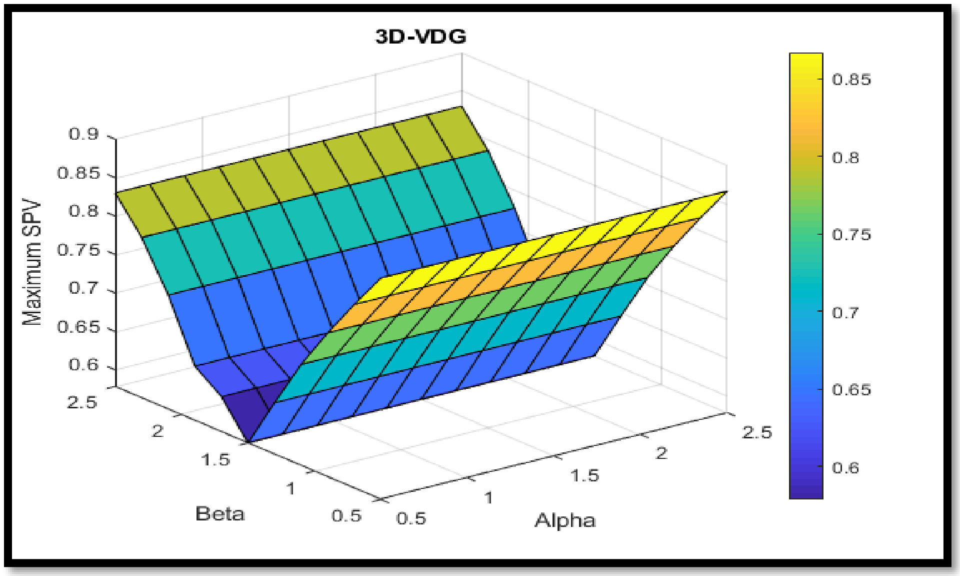

Figure 4. 3D-VDG At Variance Ratio 1 for VKM CCD (1, 2)

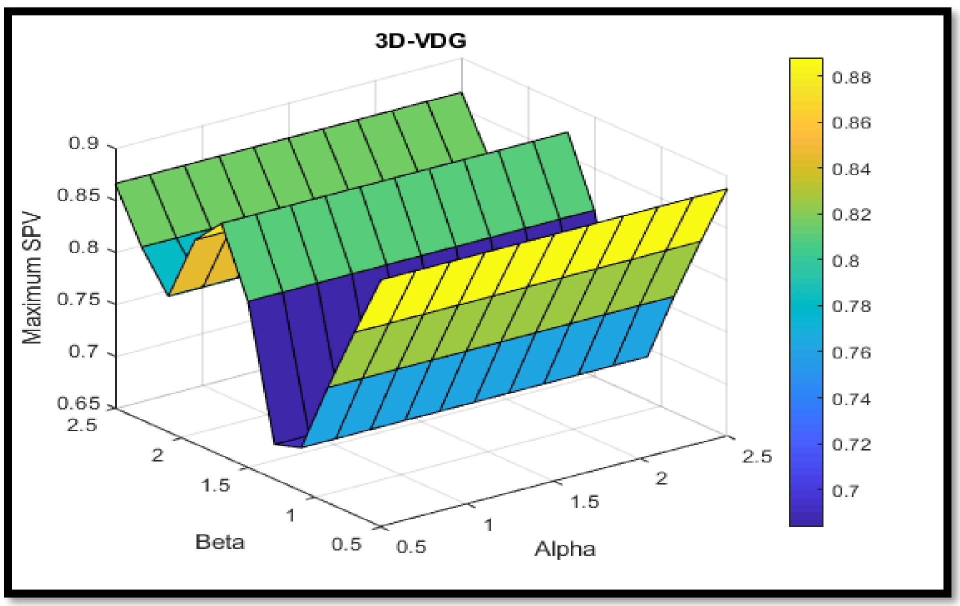

Figure 5. 3D-VDG at Variance Ratio 5 for VKM CCD (1, 2)

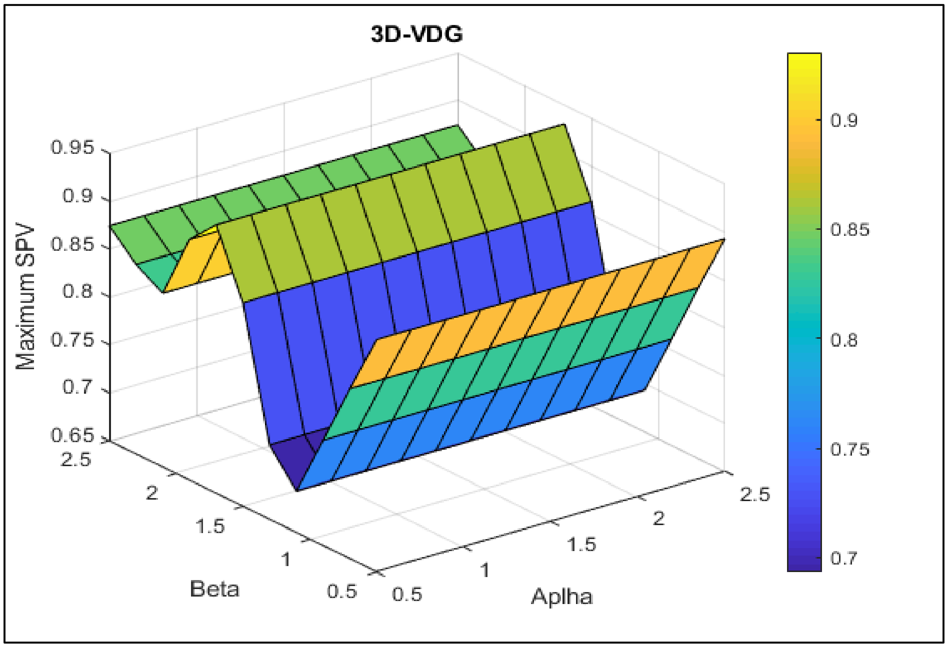

Figure 6: 3D-VDG At Variance Ratio 10 for VKM CCD (1, 2)

In three-factor VKM CCD (1, 2) shown by graphically representation on the basis of (FDS), Contour plots and 3D-suface are constructed for different values of α/β and fixed variance ratio (1,5,10), of designs points which are shown in figures (7, 8 & 9) respectively. In Fraction Design Space plot (FDS) it shows the area of design space plot having mean standard error equal to specified value. The standard error is 0.57 on the 29% in figure (9). The contour Plot for VKM CCD (1, 1) is shown in figures (10) have seven contours (0.4, 0.4, 0.4, 0.4, 0.5, 0.6 and 0.7). The 3D surface is shown in figures (11) and red dots represents the coordinates of design points, which range from 0.5 to 2.5 in coded factors units.

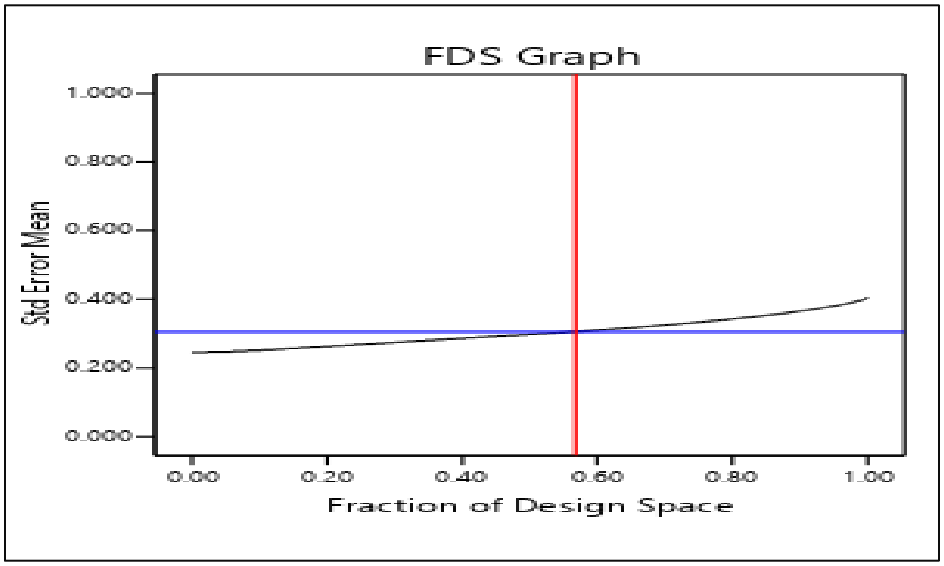

Figure 7: FDS Graph for VKM CCD (1, 2)

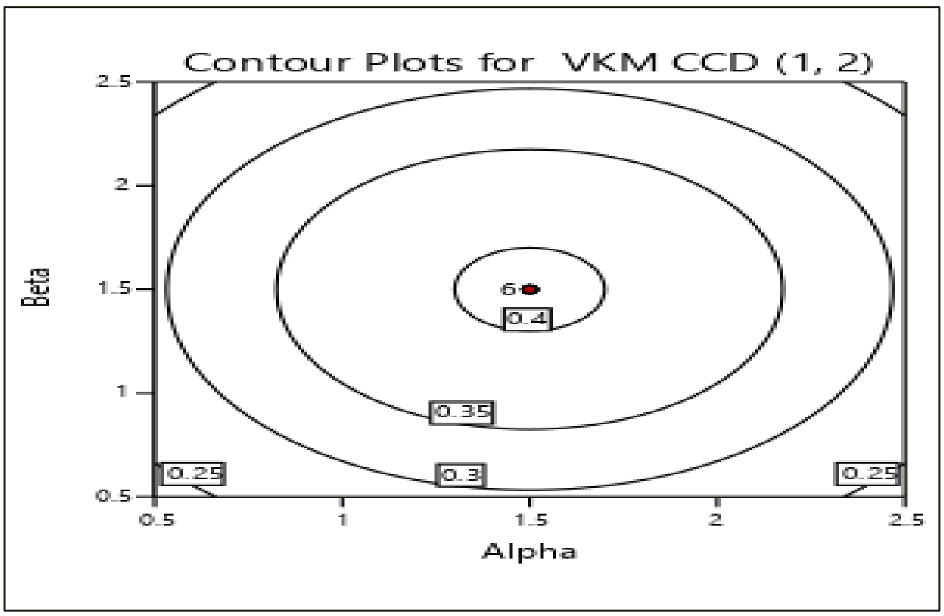

Figure 8: Contour Plots for VKM CCD (1, 2)

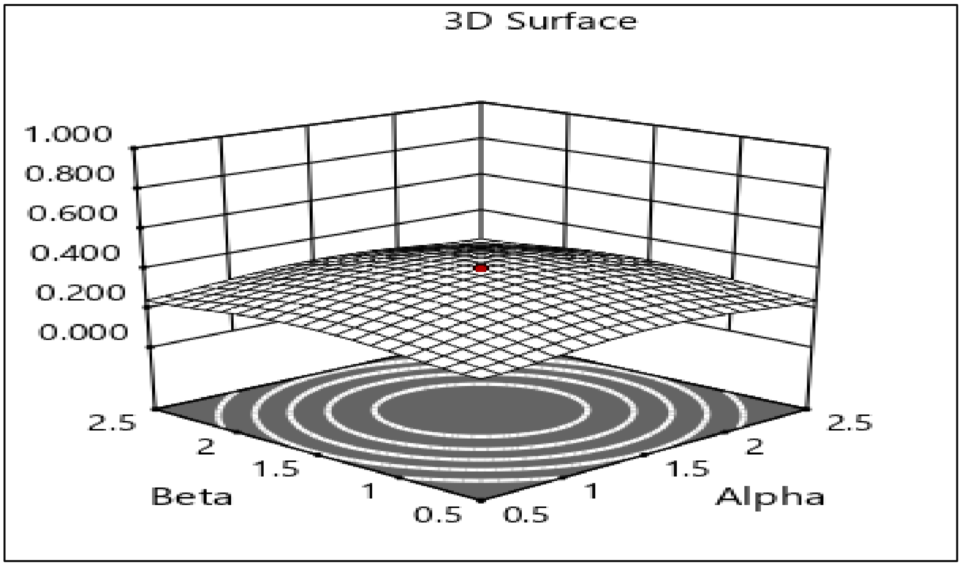

Figure 9: 3D- Surface for VKM CCD (1, 2)

4.6 Applications of Prediction Capability in Commerce and Industry

Prediction of future results is an important objective in regression analysis and analysis of variance. The studies based on experimental data in commerce and industry have many applications regarding prediction capability of the used designs as well as the models. Few of them are mentioned here:

- E-commerce brands and retailers wish to predict loyal customers when different marketing campaigns are applied to study the customer's perception
- Predicting stock market prices and other financial indicators is an important practice.
- Forecasting markets for strategic planning, providing a basis for financial institutions and governments to direct investments, and companies developing contingency plans.

5. Conclusion

The G-efficiency criterion and the variance dispersion graph are useful measures for evaluating competing designs. The VDG is a useful device for visualizing different values of the scaled prediction variance for extraordinary designs and its region in the graph space. Fraction of Design Space (FDS) methods are complementary to current VDG techniques. FDS focuses on how it predicts any section of the design space. It offers the phase of the plan house which is much less than or equal to a predefined SPV value. The FDS plot represents the graph cumulative fraction for every SPV in the design space. It lets in assessment of usual minimal and maximal values of SPVs of one-of-a-kind designs. One can see the approximate G-efficiency for that design with the specific model directly from the FDS plot. For G-efficiency against single missing observation we conclude that when the variance ratio is different [1, 5, 10] and the designs points α/β Based on the data in table (9), the following are probable outcomes are given below on different designs factors. Compute the relative loss of G-efficiency to find the robustness of VKM CCD in which study the all-sub classes such as Two-Factor (1, 1), Three-factors [(1, 2), (2, 1)], Four-Factors [(1,3), (2, 2), (3,1)], Five-Factors [(1,4), (2,3), (3, 2)] and Six-Factor [(2,4) and (3, 3)] robust VKM CCD. Over-all summary of Robust VKM CCDs against loss due to missing observation to observed the efficient designs on the diverse values of alpha/ beta and fixed variance ratio on each design points.

Table 7: Summary of Robust VKM CCDs against Missing Single Observation

w	**K**	**Parameters of Robust design**		**Minimax Loss**	**Comments**
		V. Ratio	α / β		
1	1	10	2.5	0.01681 (Factorial point)	Relative to G-efficient
1	2	10	1.3	0.00634 (WP axial point)	Relative to G-efficient
1	3	10	1.9	0.00174 (Center point)	Relative to G-efficient
1	4	10	1.9	0.00163 (Center point)	Relative to G-efficient
2	1	10	2.5	0.01472 (Subplot axial point)	Relative to G-efficient
2	2	1	0.5	0.00009 (Factorial point)	Relative to G-efficient
2	3	10	0.5	0.00855 (Factorial point)	Relative to G-efficient
2	4	10	2.1	0.00148 (WP axial point)	Relative to G-efficient
3	1	10	2.3	0.01959 (Subplot axial point)	Relative to G-efficient
3	2	10	2.5	0.00706 (Center point)	Relative to G-efficient
3	3	10	0.5	0.0086 (Center point)	Relative to G-efficient

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